

# Detecting Large-Scale Collusion in Procurement Auctions\*

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## Abstract

This paper documents evidence of widespread collusion among construction firms using a novel dataset covering most of the construction projects procured by the Japanese national government. By examining rebids that occur for auctions when all (initial) bids fail to meet the secret reserve price, we identify collusion using ideas similar to regression discontinuity. We identify about 1,000 firms whose conduct is inconsistent with competitive behavior. These bidders were awarded about 7,600 projects, or close to one fifth of the total number of projects in our sample. The value of these projects totals about \$8.6 billion. Scaling up our estimates by the size of total public construction spending in Japan, our results imply that about 0.85% of GDP, or 4% of total national investment, is affected by collusive activity by construction firms and that collusion increases government spending by about \$3.4 billion per year, or 0.4% of the total tax revenue.

Key words: Collusion, Procurement Auctions, Antitrust

JEL classification: D44, H57, K21, L12

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# 1 Introduction

One of the central themes of competition policy is to deter, detect, and punish collusion. While there is almost universal agreement among economists that collusion among firms is socially undesirable, firms often have private incentives to engage in collusive behavior absent regulatory sanctions. Therefore, it is crucial to ensure that the antitrust agencies have the authority and the resources to detect and punish collusion in order to promote competition among firms. To the extent that collusive activities remain undetected or unpunished, collusion may become the norm rather than the exception, with potentially large detrimental effects on the economy.

In this paper, we propose a way of detecting collusion in procurement auctions using bid pattern in reauctions. In procurement auctions, it is quite common for government agencies to reauction projects after an unsuccessful auction in which no bid meets the secret reserve price. Our method of detecting collusion focuses on such projects. We then apply our method to a large dataset of procurement auctions in Japan. Our novel dataset, which covers April 2003 through December 2006, accounts for most of the construction projects procured by Japan's national government during this period. Our data contain more than 40,000 projects, about 20% of which are put out for a bid multiple times.

Applying our method of detection to the sample of Japanese procurement auctions, we find evidence of widespread collusion that persists across regions, across types of construction projects and across time. We find a total of about 1,000 construction firms for which we reject the null hypothesis of competitive behavior. This is in contrast to the fact that the Japan Fair Trade Commission (JFTC) brought only four collusion cases (against a total of 92 firms) in connection with the procurement projects in our sample. The number of auctions won by the firms that we identify as uncompetitive totals 7,600, or close to one fifth of the total number of auctions in our sample. The total award amount of these auctions is about \$8.6 billion. We estimate that, absent collusion by these firms, taxpayers could have saved about \$721 million.

The spread of collusion that we find among construction firms may have economy-wide significance. The total value of public construction projects in Japan (which includes projects procured by both local and national governments) is about \$200 billion per year, or about 4% of Japan's GDP. While our dataset accounts only for public construction projects procured at the national level, firms that we identify as uncompetitive are also active in the local procurement auctions. If we simply scale up our estimates by the size of total public

construction spending, our results imply that collusive activity by construction firms affects about 0.85% of GDP, or 4% of total national investment.<sup>1</sup> If we similarly scale up the effect on taxpayers, we find that collusion increases government spending by about \$3.4 billion per year, or about 0.4% of total tax revenue.

The scale of collusion that we document in the paper also highlights the importance of rigorous enforcement of competition policy for the broader economy. Our findings lend support to the view that, absent competition policy, collusion can be widespread and affect a significant portion of an economy – as opposed to the view that collusion occurs sporadically and only under a limited set of circumstances.<sup>2</sup> Our results seem to indicate that entrusting the antitrust agencies with the authority and resources to detect and punish collusion have important aggregate-level implications.<sup>3</sup>

In order to illustrate our approach for detecting collusion, we begin by describing a procurement auction in New York discussed at some length in Porter and Zona (1993). In February 1983, the New York State Department of Transportation (DoT) held a procurement auction for resurfacing 0.8 miles of road. The lowest bid in the auction was \$4 million, and the DoT decided not to award the contract because the bid was deemed too high relative to its own cost estimates. The project was put up for a reauction in May 1983 in which all the bidders from the initial auction participated. The lowest bid in the reauction was 20% higher than in the initial auction, submitted by the previous low bidder. Again, the contract was not awarded. The DoT held a third auction in February 1984, with the same set of bidders as in the initial auction. The lowest bid in the third auction was 10% higher than the second time, again submitted by the same bidder. The DoT apparently thought this was suspicious: “It is notable that the same firm submitted the low bid in each of the auctions. Because of the unusual bidding patterns, the contract was not awarded through 1987.” (Porter and Zona, 1993, p. 523)

As this example illustrates, it is quite intuitive that a pattern of bidding in which the

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<sup>1</sup>If we scale up our estimates by the size of total public procurement (as opposed to public construction spending), the effects will be even larger. In Japan, total public procurement accounts for approximately 13% in 2008 (OECD, 2011). Public procurement accounts for about 15% of GDP worldwide, and about 20% of GDP in OECD countries (OECD, 2011).

<sup>2</sup>Porter (2005), for example, expresses a view that is close to the former: “In any market, firms have an incentive to coordinate their decisions and increase their collective profits by restricting output and raising market prices.” For the latter view, see, e.g., Schmalensee (1987) and his description of Demsetz (1973) and (1974). In his description of the Differential Efficiency Hypothesis, Schmalensee (1987) writes, “Effective collusion is rare or nonexistent.” See also Shleifer (2005), p440.

<sup>3</sup>There is also a debate on whether or not competition policy is effective at increasing total factor productivity (see, e.g., Buccirossi et al. 2012).

same bidder submits the lowest bid multiple times suggests collusion. If there is a bidding ring and the project has been preallocated to one of its members, the designated winner can be expected to submit the lowest bid in the initial auction as well as in the reauctions. The detection method we propose in this paper is based on this intuition.

While the preceding argument has intuitive appeal, the key challenge in putting this argument into practice is to control for inherent cost differences across bidders. If one bidder has sufficiently low costs relative to the rest of the bidders, the lowest bidder in the initial auction may bid the lowest in subsequent reauctions even under competition. The contribution of our paper is to operationalize the preceding argument in a way that allows us to differentiate between persistence in the identity of the lowest bidder generated by inherent cost differences and persistence generated by collusive agreements. Our method is based on ideas similar to regression discontinuity: We focus on lettings with reauctions in which, in the initial auction, the second-lowest bidder loses to the lowest bidder by a very small margin. If the two lowest bids are sufficiently close, which bidder turns out to be the lowest/second-lowest bidder in the initial auction is as good as random. Then, the two bidders can be thought of as having the same costs, on average. For these lettings, the two bidders are as likely to be the lowest bidder in reauctions as the other under competition, while collusion would still predict persistence. By focusing on lettings in which the second-lowest bidder narrowly loses to the lowest bidder in the initial auction, we can separate persistence due to competition from persistence due to collusion.

Based on this idea, we present a series of simple graphical analyses as well as two formal tests of collusion that are motivated by the graphical analyses. The first test is a test of optimality of the bidding strategies. It captures the idea that a pattern of bidding in which bidders who are outbid in the initial auction are consistently outbid in the reauction as well – even when the bidders are barely outbid in the initial auction – is inconsistent with optimal behavior. Our test statistic evaluates the extent to which the expected profits of these bidders can be improved upon by playing more aggressive strategies in the reauction. Although we do not observe bidder costs, we can bound the costs of each bidder by the bid it submits in subsequent reauctions as long as the bidder does not bid below its costs. Given that the time between the initial auction and all subsequent auctions is very short in our setting, using subsequent bids to bound costs seems reasonable. This allows us to bound the bidders' counterfactual profits from playing alternative strategies without having to fully characterize the equilibrium, which we have found very difficult to do (See Haile and Tamer (2003) for a similar approach.).

Our second test is based on the smoothness of the distribution of the bid difference in the reauction. To be precise, let  $i(1)$  and  $i(2)$  denote the lowest bidder and the second-lowest bidder in the initial auction. To the extent that neither bidder knows how the other bidder will bid in the reauction, the difference between  $i(1)$ 's bid and  $i(2)$ 's bid in the reauction will have a smooth distribution. However, if  $i(1)$  and  $i(2)$  are part of a bidding ring in which  $i(1)$  is the predetermined winner and everybody else has agreed to bid higher than  $i(1)$  in order to yield to  $i(1)$ , the difference between  $i(2)$ 's bid and  $i(1)$ 's bid in the reauction will be positive, and its distribution may exhibit a point of discontinuity at zero. Our test formalizes the notion that the discontinuity is indicative of collusion. Aside from requiring different assumptions than the first test, the second test can be applied to each firm because it is less data intensive than the first.

The approach we propose in this paper may be useful to law enforcement agencies in a variety of settings. First, having a predetermined winner is quite a common feature of bidding rings. While bidding rings can be organized in a variety of ways – depending on whether or not members can engage in side-payments, explicit communication, etc. – it is common for bidding rings to pick a predetermined winner among its members beforehand and reduce competitive pressure in the actual auction.<sup>4</sup> Preallocating the project to a designated bidder can also have significant cost advantages from the bidding ring's perspective, especially in procurement auctions. Procurement auctions typically involve substantial bidding costs due to the need for estimating the costs of the project beforehand – costs that are often borne only by the designated winner in a bidding ring.<sup>5</sup> That is, the non-designated bidders can avoid having to pay the project's estimation costs. One important implication of this is that changing the designated winner between the initial auction and the reauction may be quite difficult for the bidding ring.

Second, reauctions are quite commonly observed in procurement settings. Examples in which the auctioneer routinely holds multiple auctions for the same object include timber auctions by the U.S. Forest Services, procurement auctions by the U.S. State DoT, and U.S. offshore gas and oil lease auctions.<sup>6</sup> Because procuring agencies typically cannot commit

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<sup>4</sup>Examples include the “great electrical conspiracy” (Smith, 1961; McAfee and McMillan, 1992), bidding ring of stamp dealers (Asker, 2010b), bidding ring of construction firms in Nassau and Suffolk counties (Porter and Zona, 1993), etc.

<sup>5</sup>For example, a study by the Japan Federation of Bar Associations (JFBA) in 2001 reports that the project estimation costs are borne only by the designated winner in many bidding rings, based on facts that became clear during criminal bid-rigging cases (JFBA, 2001).

<sup>6</sup>See McAfee and Vincent (1997) for a description of reauctions in U.S. Forest Service timber sales, Ji and Li (2008) for DoT auctions in Indiana and Porter (1995) for offshore gas and lease auctions. Ji and Li (2008)

never to try to reacquire the same object when bids fail to meet the reserve price, rebidding is a very common practice in procurement auctions.

Lastly, our approach is quite simple, requiring only bidding data. Much of our analyses does not rely on parametric assumptions on the primitives of the model.

## 1.1 Related Literature

This paper contributes to the empirical literature on detecting collusion in auctions.<sup>7</sup> Existing empirical studies of collusion tend to take advantage of known episodes of cartel activity: e.g., paving in highway construction in Nassau and Suffolk counties (Porter and Zona, 1993); school milk in Ohio (Porter and Zona, 1999); school milk in Florida and Texas (Pesendorfer, 2000); and collectible stamps in North America (Asker, 2010b). While none of our analysis requires information on known bidding rings, it is still useful to study the bidding behavior of known cartels for validation purposes. We do this in Section 5 for the four known bidding cartels that were prosecuted by the JFTC in connection to our sample.

Another strand of literature tests for collusion in the absence of any prior knowledge of bidder conduct. Examples include bidding in seal coat contracts in three states in the U.S. Midwest (Bajari and Ye, 1999); U.S. Forest Service timber sales (Baldwin, Marshall and Richard, 1997; Athey, Levin and Seira, 2011); offshore gas and oil lease (Hendricks and Porter, 1988; Haile, Hendricks, Porter and Onuma, 2013); roadwork contracts in Italy (Conley and Decaloris, 2013); Libor (Abrantes-Metz et. al., 2010; Snider and Youle, 2012); and public-works consulting in Japan (Ishii, 2009). Ishii (2009) studies 175 auctions of design consultant contracts in Naha, Okinawa and analyzes how exchange of favors can explain the winner of the auctions. While her identification is based on bid patterns across auctions of different projects, our identification strategy focuses on how bidders bid across multiple auctions of a given project. Our study also looks at most of the construction projects procured by the national government, whereas she studies a specific local market.<sup>8</sup>

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examine DoT auctions in which the auctioneer sets a secret reserve price, and there are multiple auctions for the same project when none of the bids meets the secret reserve price. In their sample, they find that about 12.5% of lettings have two rounds of bidding. In his analysis of wildcat tracts, Porter (1995) reports: “A total of 233 high bids, or 10 percent were rejected on these tracts. On the tracts with rejected bids, 47 percent were subsequently reoffered.”

<sup>7</sup>For a brief survey, see Asker (2010a). For a more comprehensive study, see, e.g., Harrington (2008) and Marshall and Marx (2012).

<sup>8</sup>For a more general overview of bidding rings among procurement firms in Japan, see McMillan (1991). See, also, Ohashi (2009), who discusses how the change in auction design in Mie prefecture affected collusion.

The paper is also related to the literature on identification and estimation of incomplete models (See, e.g., Tamer (2011) for a survey.), and in particular, to the work of Haile and Tamer (2003). In Haile and Tamer (2003), they partially identify the distribution of bidders' values in English auctions using the restriction that bidders do not bid above their values. Similarly, we use the idea that the costs of bidders can be bounded above by their bids in reactions for conducting a test of collusion. The bounds on costs allow us to put bounds on the profits from playing alternative bidding strategies.

Finally, this paper is related to the literature on sequential auctions. McAfee and Vincent (1997) study the problem of a seller who can post a reserve price but cannot commit never to attempt to resell an object if it fails to sell. In a recent paper, Skreta (2013) solves for the seller's optimal mechanism with no commitment and shows that multiple rounds of either first- or second-price auctions with optimally chosen reserve prices maximize the seller's revenue when the bidders are ex-ante identical. The auctions in our dataset have the feature that the seller cannot commit never to resell but can commit to the same secret reserve price. Ji and Li (2008) structurally estimate a model of multi-round procurement auctions with a secret reserve price using data on procurement auctions organized by the Indiana DoT. The Indiana DoT also maintains the same secret reserve price throughout the multiple rounds as in our setting. Ji and Li (2008) recover the private cost distributions of the bidders assuming that the observed bids are competitive.

## 2 Institutional Background

**Auction Mechanism** The auction format used in our data is a first-price sealed-bid (FPSB) auction with rebidding. The auction mechanism is exactly the same as the standard FPSB auction as long as the lowest bid is below the secret reserve price, in which case, the lowest bidder becomes the winner with a price equal to the lowest bid, and the auction ends. If none of the bids is below the reserve price, however, the buyer reveals the lowest bid to all the bidders and solicits a second round of bids. The buyer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are not revealed). The second-round bidding typically takes place 30 minutes after the initial round, with the same set of bidders and the same secret reserve price.<sup>9</sup> This means that when bidding in the second round, the bidders know that the secret reserve price is lower

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<sup>9</sup>See, e.g., Bidding Guidelines of Ministry of Land, Infrastructure, and Transportation, Chugoku Regional Developing Bureau.

than the lowest first-round bid.

The second round proceeds in the same manner as the initial round; if the lowest bid is below the reserve price, the auction ends, and the lowest bidder wins. Otherwise, the buyer reveals the lowest second-round bid to the bidders, and the auction goes to the third round. The third round is the final round. If no bid meets the reserve price in the third round, bilateral negotiation takes place between the buyer and the lowest third-round bidder. The same secret reserve price is used in all three rounds.

Bidding takes place online in all three rounds, and the identity of the bidders is not public at the time of bidding. The reserve price, the identity of the bidders, and all the bids in each round are made public after the auction ends.

**Bidder Participation** As is the case in many countries, participation in procurement auctions in Japan is not fully open. A contractor that wishes to participate must first go through screening to be pre-qualified. Because pre-qualification occurs at the regional level, a contractor needs to be pre-qualified for each region in which it wishes to bid on projects.<sup>10</sup>

In addition to pre-qualification, there may be additional restrictions on participation. Depending on how restrictive these are, the auctions can be divided into four categories. The first and the second categories are the most restrictive, with participation by government invitation. In these two categories, the government typically invites ten bidders from the pool of pre-qualified contractors. The difference between the two categories is that in the second category, the government chooses bidders based on contractors' preferences over project type, project location, etc., submitted by the contractors in advance, while in the first category, the invited bidders are chosen without consideration of contractors' preferences.<sup>11</sup> In both categories, the government has considerable discretion over who is invited.<sup>12</sup>

The third and fourth categories are less restrictive. The set of potential bidders is still restricted to the pool of pre-qualified contractors, but any pre-qualified contractor can participate in the bidding. The difference between the third and fourth categories is that in the third category, the government reserves the right to exclude potential bidders from partici-

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<sup>10</sup>Our dataset is divided into nine regions (Hokkaido, Tohoku, Kanto, Hokuriku, Chubu, Kansai, Chugoku, Shikoku, and Kyushu).

<sup>11</sup>Each pre-qualified contractor submits a form to the government every two years to express its preferences over the type and location of projects it wishes to bid on.

<sup>12</sup>See, e.g., Ohno (2003).



pating in the auction under certain conditions.

**Collusive Behavior** In principle, bidding rings can be organized in a variety of ways, depending on whether or not members engage in side payments, whether explicit communication between the members is feasible, etc. Whatever the exact arrangement, however, a very common feature of bidding rings is that the ring picks a predetermined winner in advance and that the rest of the ring members help that predetermined winner win. The existing evidence indicates that bidding rings in the construction industry in Japan are organized in this manner.<sup>13</sup>

Two other documented features of prosecuted bidding rings in the construction industry are worth mentioning: First, the designated winner alone typically incurs the cost of estimating the project cost.<sup>14</sup> Estimating the project cost can be quite expensive, and the non-designated bidders typically avoid incurring this cost.<sup>15</sup> Note that this makes it risky for a non-designated bidder to accidentally win the auction. The second feature is that the designated winner of a bidding ring would often communicate to other members in advance how it would bid in each of the three rounds (as opposed to communicating how it would bid in just the first round).<sup>16</sup>

### 3 Data

We use a novel dataset of auctions for public construction projects obtained from the Ministry of Land, Infrastructure and Transportation, the largest single procurement buyer in Japan. The dataset spans April 2003 through December 2006 and covers most of the construction works auctioned by the Japanese national government during this period. After dropping scoring auctions, unit-price auctions, and those with missing or mistakenly recorded entries, we are left with 42,561 auctions with a total award amount of more than

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<sup>13</sup>See, for example, a report issued by the JFBA, which studies criminal bid-rigging cases (JFBA, 2001).

<sup>14</sup>See, e.g., the criminal bid-rigging case regarding the construction of a sewage system in Hisai city (Tsu District Court, No. 165 (Wa), 1997); the bid-rigging case regarding the construction of a waste incineration plant in Nagoya city (Nagoya District Court, No. 1903 (Wa), 1995); etc. Based on facts that became clear in these cases, the JFBA concludes that the project estimation costs are borne only by the designated winner in many bidding rings.(JFBA (2001), p20)

<sup>15</sup>Estimating the project cost involves understanding the specifications of the project, assessing the quantity and quality of materials required, negotiating prices for construction material and arranging for available subcontractors. These costs are often quite substantial.

<sup>16</sup>See JFBA (2001), p19 and JFTC Ruling #27 (2010), pp.10-11.

Concluding Round	(R)eserve Yen M. (1)	(W)inbid Yen M. (2)	(W)/(R) (3)	Lowest bid / Reserve			# Bidders (7)	N (8)
				Round 1 (4)	Round 2 (5)	Round 3 (6)		
1	103.459 (246.55)	97.011 (234.01)	0.927 (0.085)	0.927 (0.085)	-	-	9.86 (2.60)	34,104 80.1%
2	81.033 (177.85)	78.526 (173.40)	0.964 (0.033)	1.056 (0.075)	0.964 (0.033)	-	9.90 (2.40)	7,207 16.9%
3	62.375 (166.45)	60.050 (157.39)	0.962 (0.035)	1.143 (0.113)	1.071 (0.089)	0.962 (0.035)	9.42 (2.25)	1,250 2.9%
All	98.455 (234.49)	92.795 (223.11)	0.934 (0.079)	0.955 (0.102)	0.980 (0.059)	0.962 (0.035)	9.86 (2.56)	42,561 100.0%

Note: The first row corresponds to the summary statistics of auctions that ended in the first round; the second row corresponds to auctions that ended in the second round; and the third row corresponds to auctions that went to the third round. The last row reports the summary statistics of all auctions. The numbers in parentheses are the standard deviations. First and second columns are in millions of yen.

Table 1: Sample Statistics

\$42 billion.<sup>17</sup>

The data include information on all bids, bidder identity, the secret reserve price, auction date, auction category (which corresponds to how restrictive bidder participation is), location of the construction site, and the type of project.<sup>18</sup> The data also contain information on whether the auction proceeded to the second round or the third round, as well as all the bids in each round. Table 1 provides summary statistics of the data. In the table, we report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)). The sample statistics are reported separately by whether the auction concluded in Round 1, Round 2, or Round 3.

In the first and second columns of the table, we find that the average reserve price of the auctions is about 98 million yen, and the average winning bid is about 93 million yen. In the third column, we find that the winning bid ranges between 92% and 97% of the reserve

<sup>17</sup>Samples with missing or mistakenly recorded entries each account for 1.3% of the entire dataset. The scoring auction data account for 15.8%.

<sup>18</sup>Construction projects are divided into 21 types of construction work, such as civil engineering, architecture, bridges, paving, dredging, painting, etc.

		Round 2				
		1	2	3	4	5+
Round 1	1	96.70%	1.61%	0.62%	0.26%	0.82%
	2	1.59%	26.62%	18.63%	13.50%	39.66%
	3	0.53%	18.81%	18.65%	13.89%	48.11%
	4	0.37%	14.24%	15.94%	15.36%	54.10%
	5+	0.13%	6.75%	9.21%	10.34%	73.56%

Note: The  $(i,j)$  element of the matrix denotes the probability that a bidder submits the  $j$ -th lowest bid in the second round conditional on submitting the  $i$ -th lowest bid in the first round. When there are ties, multiple bidders are assigned to the same rank. The number of auctions is 8,089.

Table 2: Rank of the Second-Round Bid by Rank of the First-Round Bid

price. In the next three columns, we report the lowest bid in each round as a fraction of the reserve price. Note that for auctions that conclude in the first round, Column (4) is equal to Column (3). For auctions that conclude in the second or third round, the numbers reported in Column (4) are higher than unity by construction. Column (8) reports the sample size. We find that 19.9% of the auctions go to the second round, and 2.9% advance to the third round.

## 4 Analysis

### 4.1 Persistence of the Identity of the Lowest Bidder

**Persistence in the Second Round** We begin our analysis by studying the extent to which the lowest bidder in the first round is also the lowest bidder in later rounds for a given auction. Recall that a typical feature of bidding rings is that there is a designated winner and that other ring members submit bids in such a way as to ensure that the designated bidder is the lowest bidder. Because, in the setting we study, the bidding rings does not know the reserve price, the ring members must make sure that the designated bidder is the lowest bidder in each successive round if the auction takes multiple rounds. This implies that we should observe persistence in the identity of the lowest bidder across rounds under bidder collusion.

In Table 2, we report how the rank of the bidders changes from the first round to the

second round for all auctions that proceed to the second round with five or more participants ( $N = 8,089$ ). The  $(i, j)$  element of the matrix corresponds to the probability that a bidder submits the  $j$ -th lowest bid in the second round, conditional on submitting the  $i$ -th lowest bid in the first round; i.e.,  $\Pr(j\text{-th lowest} | i\text{-th lowest})$ . Thus, the diagonal elements correspond to the probability that a given bidder remains in the same rank in both rounds. Note that the horizontal sum of the probabilities is one.

What is striking about this table is the probability in the  $(1, 1)$  cell. We find that in 96.70% of the auctions, the lowest bidder in the first round is still the lowest bidder in the second round. The flip side of this is that if a bidder is not the lowest bidder in the first round, the bidder is almost never the lowest bidder in the second round. For example, the conditional probability that a second-lowest bidder in Round 1 becomes the lowest bidder in Round 2 is only 1.59%. Note, also, that the diagonal elements other than the  $(1, 1)$  element are much smaller: the probability that the second-lowest bidder in the first round remains the second-lowest bidder is just 26.62%. There is very strong persistence in the identity of the lowest bidder, but not necessarily for other positions.

In order to illustrate this point further, we examine more closely how the three lowest bidders in the first round behave in the second round. In what follows, we let  $i(k)$  denote the identity of the bidder who submits the  $k$ -th lowest bid in Round 1. We also denote the (normalized) bid of bidder  $i(k)$  in round  $t$  by  $b_{i(k)}^t$ . Because there is considerable variation in project size, we work with the normalized bids by dividing the actual bids by the reserve price of the auction. Hence,  $b_{i(1)}^2$ , for example, denotes the second-round bid of the first-round lowest bidder as a percentage of the reserve price.

In the top left panel of Figure 1, we plot the histogram of  $\Delta_{12}^2 \equiv b_{i(2)}^2 - b_{i(1)}^2$  for the set of auctions that go to the second round. That is, we plot the difference in the (normalized) second-round bids of  $i(1)$  and  $i(2)$ .<sup>19</sup> Note that almost all of the mass lies to the right of zero, which confirms what we report in Table 2: a flip in the ordering between the lowest and the second-lowest bidders almost never happens across rounds. In the top right panel of Figure 1, we plot the histogram of  $\Delta_{23}^2 \equiv b_{i(3)}^2 - b_{i(2)}^2$ , i.e., the difference in the normalized rebids of  $i(2)$  and  $i(3)$ , for the set of auctions that go to the second round. In stark contrast to the left panel, the shape of the histogram for  $\Delta_{23}^2$  is quite symmetric around zero. This implies that the ranking between  $i(2)$  and  $i(3)$  flips in the second round with almost 50% probability. This also seems consistent with our previous finding that there is much less

<sup>19</sup>The sample sizes are different between the top left and the top right panels because in some auctions,  $i(1)$  or  $i(3)$  does not bid in the second round.

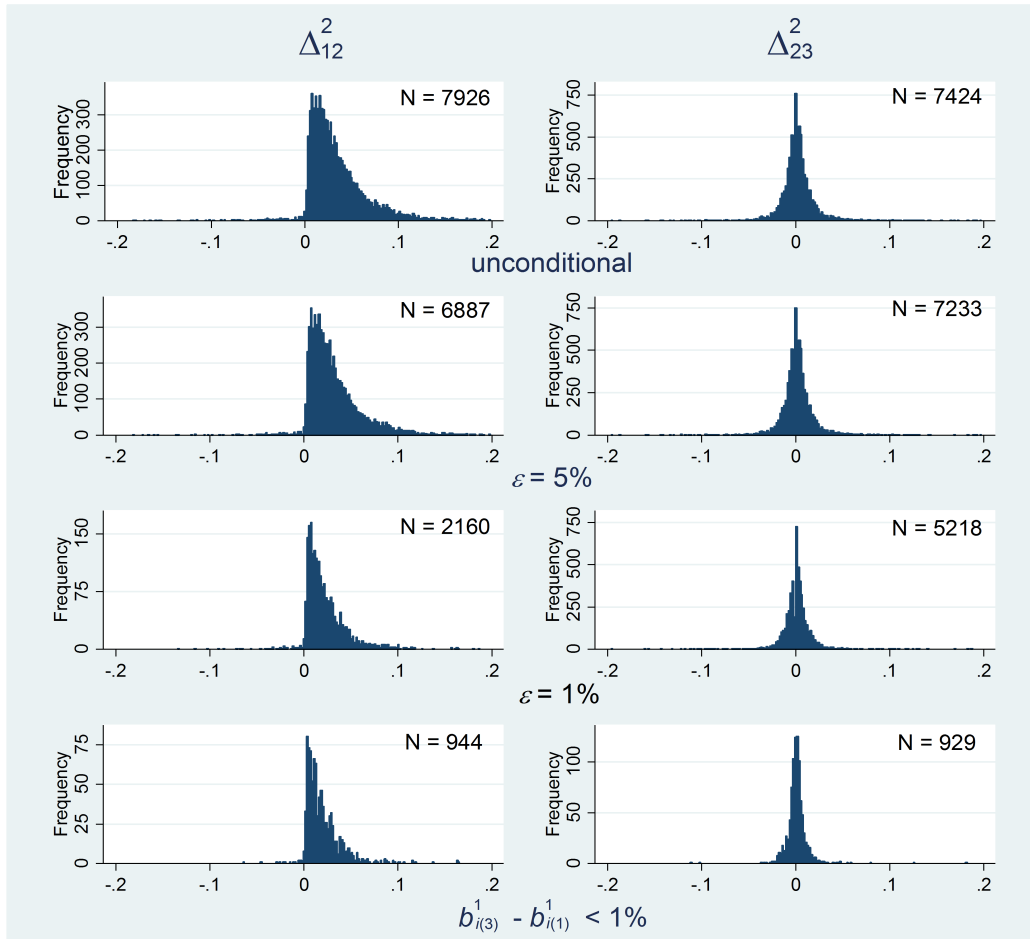


Figure 1: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panels) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panels). The first row is the histogram for the set of auctions that reach the second stage; and  $i(1)$  and  $i(2)$  (or  $i(2)$  and  $i(3)$ ) submit valid bids in the second round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small. Note that the sample sizes are different between the top left and the top right panels because in some auctions,  $i(1)$  or  $i(3)$  does not bid in the second round. Similarly for the bottom left and right panels.

persistence in the ranking for the second and third places.

So far, the results that we have presented correspond to all of the auctions that proceed to the second round. However, it is possible that our results are driven by inherent differences among firms with respect to costs, risk attitude, beliefs over the reserve price, etc. For instance, if there are significant cost differences between  $i(1)$  and all of the other bidders, our results may be generated by competitive bidding. In order to rule out this possibility,

we perform the same analysis by conditioning on the set of auctions in which the first-round bids are close to each other. The idea is that if, for example, the first-round bids of  $i(1)$  and  $i(2)$  are sufficiently close (i.e.,  $b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon$  for some small  $\varepsilon$ ), there should be little inherent difference among them, on average. In fact, if  $\varepsilon$  is small enough, which bidder turns out to be the lowest/second-lowest bidder in the first round is as good as random. In this case,  $i(1)$  and  $i(2)$  should be interchangeable in terms of costs, risk attitude, beliefs over the reserve price, etc.

In the second row of Figure 1, we plot  $\Delta_{12}^2$  and  $\Delta_{23}^2$  for the subset of auctions for which the bid differences in the first round are within 5% of the reserve price.<sup>20</sup> In particular, we plot the histogram of  $\Delta_{12}^2$  for the set of auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$  in the left panel and the histogram of  $\Delta_{23}^2$  for the set of auctions with  $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$  in the right panel. Note that the shape of the distribution of  $\Delta_{12}^2$  in the left panel is still very skewed and asymmetric around zero, while the distribution of  $\Delta_{23}^2$  in the right panel remains symmetric around zero. The third row plots the distribution of  $\Delta_{12}^2$  and  $\Delta_{23}^2$ , but now conditioning on auctions with  $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$  and  $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$ , respectively. Lastly, the bottom row shows the distribution of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  conditional on the event that the three lowest bids in the first round are *all* within 1% of the reserve price,  $b_{i(3)}^1 - b_{i(1)}^1 < 0.01$ .<sup>21</sup> Note that shapes of the distributions remain similar across rows. In particular, the fact that the distribution of  $\Delta_{23}^2$  is symmetric around zero and very similar across rows suggests that cost differences between bidders do not seem to play a large role: if cost differences were driving the skewed bid pattern for  $\Delta_{12}^2$  in the left panel, we should also expect to see a distribution of  $\Delta_{23}^2$  that is skewed to the right of zero. Taken together, Figure 1 suggests that it is not differences in costs, etc. that are driving the persistence in the identity of the lowest bidder.

In the Online Appendix, we explore whether the distributions of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  exhibit similar patterns when we condition the sample by various auction characteristics, such as region, auction category, project type, and year. We find that the distributions of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  often look very similar to those shown in Figure 1: The distribution of  $\Delta_{12}^2$  is skewed to the right and displays what appears to be a discontinuity at zero, while the distribution

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<sup>20</sup>The sample sizes are different between the two panels in the second and third rows because there are more auctions in which  $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$  than auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$ . The difference in the sample sizes in the first and fourth rows is due to the fact that in some auctions,  $i(1)$  or  $i(3)$  does not bid in the second round.

<sup>21</sup>Note that  $b_{i(3)}^1 \geq b_{i(2)}^1 \geq b_{i(1)}^1$ , by construction. Hence,  $b_{i(3)}^1 - b_{i(1)}^1 < 0.01$  implies  $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$  and  $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$ .

of  $\Delta_{23}^2$  is symmetric around zero. In the Online Appendix, we also plot the second-round bid difference between  $i(1)$  and  $i(2)$  as well as the bid difference between  $i(2)$  and  $i(3)$  without normalizing the bids by the reserve price. The graphs also appear similar to Figure 1.

**Information Advantage of  $i(2)$**  Recall from Section 2 that, at the end of each round, the lowest bid is announced, but none of the other bids are. This means that while  $i(1)$  learns only that it was the lowest bidder in the first round,  $i(2)$  learns exactly what the lowest bidder bid in the first round in addition to what it bid itself. This implies that, conditional on the two lowest bids being very close to each other,  $i(2)$  has an information advantage over  $i(1)$  in the second round. To see this, consider the case in which  $i(1)$  and  $i(2)$  bid almost exactly the same amount, say  $\$Z$ . The information revealed to  $i(1)$  at the end of the first round is that  $\$Z$  is the lowest bid and that it bid the lowest. The information revealed to  $i(2)$ , on the other hand, is that  $\$Z$  is the lowest bid and that (at least) one other firm beside itself bid  $\$Z$ . Clearly,  $i(2)$  has a bigger information set at the end of the first round.

So far, we have documented that the ordering between  $i(1)$  and  $i(2)$  is very persistent across rounds, while the ordering between  $i(2)$  and  $i(3)$  is not. Given  $i(2)$ 's information advantage, however, the fact that the ordering between  $i(1)$  and  $i(2)$  does not change is even more surprising. Once we condition on auctions in which  $i(1)$  and  $i(2)$  bid close to each other in Round 1,  $i(2)$  should be aware that, by bidding a little more aggressively, it can beat  $i(1)$  in the next round with high probability. Hence, given the informational advantage of  $i(2)$ , we would normally expect the order of  $i(1)$  and  $i(2)$  to flip more, and not less, frequently than 50% under competitive behavior. Hence, the persistence in the identity of the lowest bidder seems at odds with competitive behavior.

**Persistence in the Third Round** For the subset of auctions that go to the third round, we can further examine whether a similar pattern continues to hold in the third round. In the top two panels of Figure 2, we plot the difference in the third-round bids of  $i(1)$  and  $i(2)$ , i.e.,  $\Delta_{12}^3 \equiv b_{i(2)}^3 - b_{i(1)}^3$  (left panel), and the difference in the third-round bids of  $i(2)$  and  $i(3)$ , i.e.,  $\Delta_{23}^3 \equiv b_{i(3)}^3 - b_{i(2)}^3$  (right panel) for all auctions that advance to the third round. In rows two to four of Figure 2, we plot the histogram conditioning on the set of auctions in which the first-round bids are relatively close. Focusing on the left panels, the second row plots  $\Delta_{12}^3$  for the set of auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 0.05$ ; the third row plots  $\Delta_{12}^3$  for

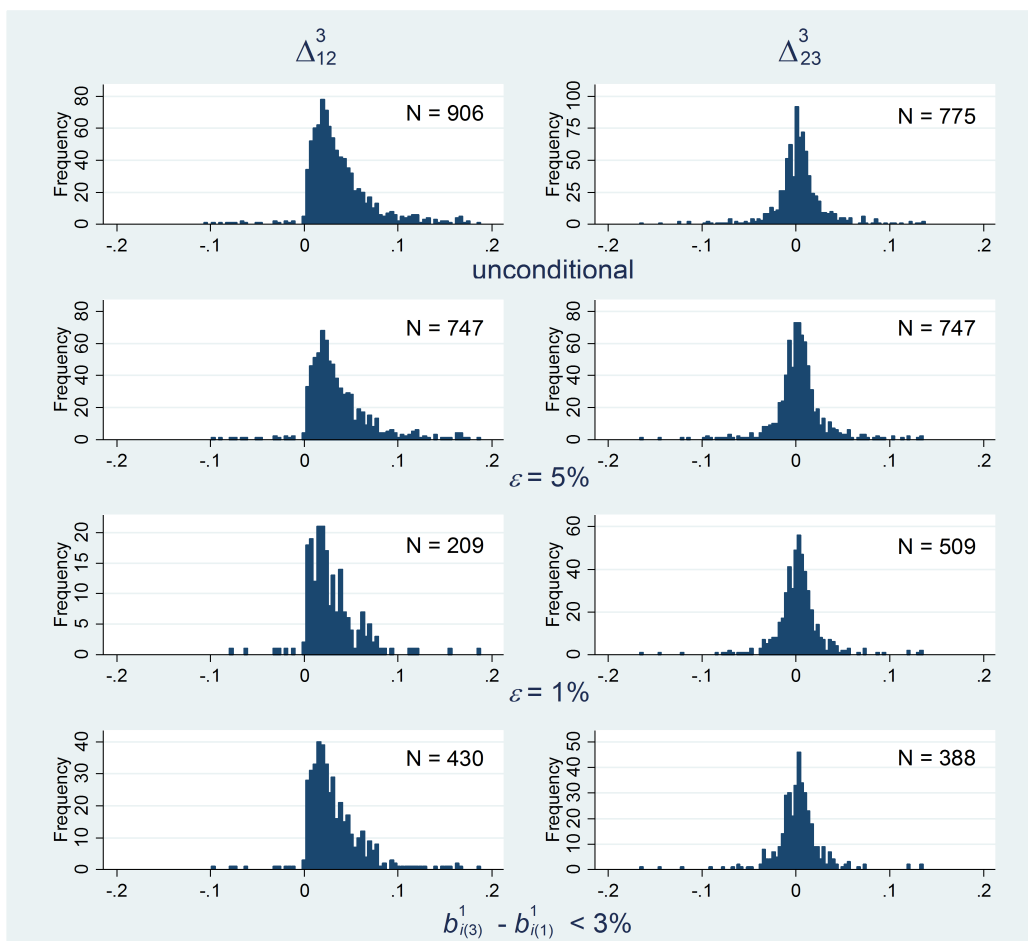


Figure 2: Difference in the Third-Round Bids of  $i(1)$  and  $i(2)$  (Left Panels) and the Difference in the Third-Round Bids of  $i(2)$  and  $i(3)$  (Right Panels). The first row corresponds to all auctions that reached the third round and  $i(1)$  and  $i(2)$  (in the case of the left panel) or  $i(2)$  and  $i(3)$  (in the case of the right panel) submitted valid bids in the third round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.

which  $b_{i(2)}^1 - b_{i(1)}^1 < 0.01$ ; and the last row plots  $\Delta_{12}^3$  for which  $b_{i(3)}^1 - b_{i(1)}^1 < 0.03$ . Similarly, the second through the fourth panels in the right column plot  $\Delta_{23}^3$  for the set of auctions in which  $b_{i(3)}^1 - b_{i(2)}^1 < 0.05$ ,  $b_{i(3)}^1 - b_{i(2)}^1 < 0.01$  and  $b_{i(3)}^1 - b_{i(1)}^1 < 0.03$ , respectively.

## 4.2 Discontinuity of the distribution of $\Delta_{12}^2$ at Zero

One striking feature of the distribution of  $\Delta_{12}^2$  (and  $\Delta_{12}^3$ ) is that there is what appears to be a discontinuous jump at exactly zero. This is in stark contrast to the distribution of  $\Delta_{23}^2$  (and



$\Delta_{23}^3$ ), which is symmetric and continuous around zero. We argue that this discontinuous jump in the distribution of  $\Delta_{12}^2$  is further evidence that the persistence cannot have been generated by competitive behavior.

Consider, first, the distribution of  $\Delta_{23}^2$  in the right panels of Figure 1. Note that, even among bidders that submit almost identical first-round bids, there is a certain amount of variance in  $\Delta_{23}^2$ . To the extent that these bids are generated under competitive behavior, this seems to indicate that for many auctions, there is a reasonable amount of idiosyncrasy among the bidders with regard to their beliefs over the distribution of the reserve price, risk preference, etc., inducing variance in the second-round bids. In other words, idiosyncratic reasons seem to induce at least a certain amount of uncertainty in the second-round bidding for many auctions, even among bidders that submit almost identical first-round bids.

Now consider the distribution of  $\Delta_{12}^2$  in the left panels of Figure 1. As long as there exists a reasonable amount of idiosyncrasy among the bidders,  $i(2)$  should outbid  $i(1)$  in the second round by a narrow margin just as often as  $i(1)$  outbids  $i(2)$  by a narrow margin. That is, there should be a similar number of observations in which  $\Delta_{12}^2 \in [-t, 0]$  and  $\Delta_{12}^2 \in [0, t]$  for small values of  $t$  – a feature which we clearly do not see in any of the histograms of the left panels of Figure 1. This is inconsistent with competitive behavior.

In fact, the discreteness exhibited in the histogram of  $\Delta_{12}^2$  at zero suggests that the bidders know exactly how the other bidders will bid in the second round. If, on the contrary,  $i(1)$  and  $i(2)$  were both uncertain about each other's bid, there should be just as many cases where  $i(2)$  won by a tiny margin as cases where  $i(2)$  lost by a tiny margin. Hence, the discontinuity of  $\Delta_{12}^2$  suggests that the bidders have prior knowledge about how each other will bid and that  $i(2)$  is deliberately losing by submitting a slightly higher bid than  $i(1)$  (rather than winning by slightly underbidding  $i(1)$ ).

Note that our findings suggest that bidding rings determine beforehand how each ring member should bid in the second round – not just how to bid in the first round. This is natural, given that a substantial fraction of auctions go to the second round and that there are only 30 minutes between rounds. This is also consistent with a court ruling in which the prosecuted ring members are described as coordinating how to bid in multiple rounds ahead of time. (See, Nagoya District Court, No. 1903 (Wa), 1995).

In Section 6, we develop a test statistic of collusive behavior that we can apply for each firm (as opposed to pooling across auctions of different firms) based on the idea discussed in this section. That is, we use the idea that the distribution of  $\Delta_{12}^2$  should not be discontinuous at zero under competitive bidding.

### 4.3 Optimality of Second-Round Bidding Strategy

Recall that there are many cases in which  $i(2)$  could have outbid  $i(1)$  in the second round by lowering its second-round bid by a tiny margin. For example, focusing on the left panel of the second row in Figure 1, we find that about 15.75% and 38.67% of the distribution lie within  $[0, 0.01]$  and  $[0, 0.02]$ , respectively. On the other hand, the fraction of the distribution that lies to the left of zero is only 1.73%. This suggests that  $i(2)$  can increase the probability of outbidding  $i(1)$  substantially by decreasing its bid only slightly, raising the question of whether  $i(2)$ 's second-round bid is optimal.<sup>22</sup>

Based on this observation, we construct a formal test of competition in this section. While we have found it very difficult to fully characterize the equilibrium under competitive bidding in our setting, we can still test whether the observed bidding pattern is consistent with a competitive equilibrium.<sup>23</sup> A necessary condition of a competitive equilibrium is that each firm bids optimally, given the strategies played by everybody else. Thus, if we can find an alternative bidding strategy for one of the bidders that yields higher expected profits for that bidder, this implies that the observed bidding patterns are inconsistent with competitive behavior. We apply this argument to the second-round bidding strategy employed by bidders who barely lose in the first round. In particular, we show that bidders who barely lose in the first round can substantially increase their expected profits by decreasing their second-round bid by a small margin.

The key idea behind our analysis in this section is that the firm's third-round bid can provide an upper bound on its costs under private values. Using this idea, we can compute a lower bound on the bidder's profits from playing an alternative bidding strategy in the second round without fully characterizing the equilibrium. This approach is similar in spirit to that of Haile and Tamer (2003), who obtain an upper bound on the value of bidders in an incomplete model of English auctions using the assumption that bidders do not bid above their value.

In what follows, we compare, for bidders that barely lose in the first round, the expected profits from using the current second-round strategy and the expected profits from using an alternative second-round strategy. The alternative strategies that we consider are of the

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<sup>22</sup>Strictly speaking,  $i(2)$  does not know that it came in second at the time of rebidding (it learns only that it came close to being first). The analysis below takes this into consideration.

<sup>23</sup>Solving for the equilibrium is difficult because the lowest bid is revealed at the end of the first round. Even for two symmetric bidders and two rounds under independent private values, there is no equilibrium in which everybody follows a symmetric increasing equilibrium in both rounds. The proof is available upon request.

form  $x\mathbf{b}_i^2$ , where  $x$  is some number less than 1 (e.g., 0.99) and  $\mathbf{b}_i^2$  is the bidder's current (unnormalized) second-round bidding strategy. Just for this section, we work with the raw bids without normalizing by the reserve price. We show below that, for a range of values of  $x$ , the expected profits actually increase.

First, let  $i$  be a bidder who bid slightly higher than  $i(1)$  in the first round. We will be more precise about the meaning of "slightly" later. Consider bidder  $i$ 's expected profits from using the current strategy,  $\mathbf{b}_i^2$ , conditional on advancing to the second round. The expected profits consist of two components: the expected profits from winning in the second round; and the expected profits from being the lowest bidder in the third round if the auction advances to the third round. We denote by  $W^2$  the event that bidder  $i$  wins in the second round and by  $W^3$  the event that bidder  $i$  is the lowest third-round bidder,

$$\begin{aligned} W^2 &= \{\mathbf{b}_i^2 < \min\{r, \min_{j \neq i} \mathbf{b}_j^2\}\} \\ W^3 &= \{\mathbf{b}_i^3 < \min_{j \neq i} \mathbf{b}_j^3 \wedge \min_j \mathbf{b}_j^2 > r\}, \end{aligned}$$

where  $r$  is the secret reserve price and  $\mathbf{b}_i^3$  is bidder  $i$ 's current third-round bidding strategy. Note that  $W^3$  includes both the event that  $\mathbf{b}_i^3$  is below  $r$  and the event that it is above  $r$ . We now express bidder  $i$ 's expected profits under  $\mathbf{b}_i^2$ :

$$\pi_i = \Pr(W^2)\mathbf{E}[\mathbf{b}_i^2 - c_i | W^2] + \Pr(W^3)\mathbf{E}[\text{profits} | W^3],$$

where  $c_i$  is bidder  $i$ 's costs. The expected profits in event  $W^3$  is either  $\mathbf{b}_i^3 - c_i$ , if  $\mathbf{b}_i^3$  is lower than  $r$ , or some number less than  $\mathbf{b}_i^3 - c_i$  (which depends on how the bilateral negotiation between bidder  $i$  and the government plays out) if  $\mathbf{b}_i^3$  is higher than  $r$ . In either case, the expected profits in event  $W^3$  are less than  $\mathbf{E}[\mathbf{b}_i^3 | W^3]$ . Thus, we can bound  $\pi_i$  from above as follows:

$$\pi_i \leq \Pr(W^2)\mathbf{E}[\mathbf{b}_i^2 - c_i | W^2] + \Pr(W^3)\mathbf{E}[\mathbf{b}_i^3 | W^3].$$

This bound is not very tight because we are setting bidder  $i$ 's costs equal to zero for event  $W^3$ .

Now consider the expected profits,  $\tilde{\pi}_i$ , from an alternative second-round bidding strategy that discounts current second-round bids by some factor  $x \in (0, 1)$ . As before,  $\tilde{\pi}_i$  consists of two components, the expected profits from the second round and the expected

profits from the third round:

$$\tilde{\pi}_i = \Pr(\tilde{W}^2) \mathbf{E}[x\mathbf{b}_i^2 - c_i | \tilde{W}^2] + \mathbf{E}[\text{third round profits}],$$

where  $\tilde{W}^2$  is the event in which bidder  $i$  wins in the second round using strategy  $x\mathbf{b}_i^2$ , i.e.,  $\{x\mathbf{b}_i^2 < \min\{r, \min_{j \neq i} \mathbf{b}_j^2\}\}$ . Because we are interested only in obtaining a lower bound for  $\tilde{\pi}_i$ , it is not necessary to specify  $\mathbf{E}[\text{third-round profits}]$  other than to say that it is nonnegative. Thus, we obtain the lower bound on  $\tilde{\pi}_i$  as follows:

$$\tilde{\pi}_i \geq \Pr(\tilde{W}^2) \mathbf{E}[x\mathbf{b}_i^2 - c_i | \tilde{W}^2]. \quad (1)$$

We now compare the change in expected profits,  $\Delta\pi$ , from bidding  $x\mathbf{b}_i^2$  instead of  $\mathbf{b}_i^2$ . Using the bounds obtained above,  $\Delta\pi$  can be bounded below as follows:

$$\begin{aligned} \Delta\pi = \tilde{\pi}_i - \pi_i &\geq \Pr(\tilde{W}^2 - W^2) \mathbf{E}[x\mathbf{b}_i^2 | \tilde{W}^2 - W^2] - \Pr(\tilde{W}^2 - W^2) \mathbf{E}[c_i | \tilde{W}^2 - W^2] \\ &\quad - \Pr(W^2) \mathbf{E}[(1-x)\mathbf{b}_i^2 | W^2] - \Pr(W^3) \mathbf{E}[\mathbf{b}_i^3 | W^3], \end{aligned} \quad (2)$$

where  $\tilde{W}^2 - W^2 = \tilde{W}^2 \cap (W^2)^C$ .<sup>24</sup> Note that  $\tilde{W}^2 - W^2$  is the event in which bidder  $i$  wins in the second round with  $x\mathbf{b}_i^2$  but not with  $\mathbf{b}_i^2$ . Because we consider  $x \in (0, 1)$ ,  $\tilde{W}^2$  is a superset of  $W^2$ , i.e.,  $\tilde{W}^2 \supset W^2$ . The potential gain from using strategy  $x\mathbf{b}_i^2$  instead of  $\mathbf{b}_i^2$  occurs in event  $\tilde{W}^2 - W^2$ , and the amount of the gain is  $(x\mathbf{b}_i^2 - c_i)$ .<sup>25</sup> The first two terms on the right-hand side of expression (2) correspond to the gain. The third term corresponds to the potential loss from using  $x\mathbf{b}_i^2$ . In event  $W^2$ , using  $x\mathbf{b}_i^2$  is less profitable than  $\mathbf{b}_i^2$  because bidder  $i$  is already winning with a bid of  $\mathbf{b}_i^2$ .

All of the terms in expression (2), except for  $\mathbf{E}[c_i | \tilde{W}^2 - W^2]$ , can be evaluated directly from the data, in the sense that sample analogues can be constructed. For example, for any given value  $x$ ,  $\mathbf{E}[x\mathbf{b}_i^2 | \tilde{W}^2 - W^2]$  can be evaluated by taking the sample average of  $x\mathbf{b}_i^2$  for auctions in which a bidder does not win in the second round, but would have won if it had bid  $x$  (e.g., 0.99) of the original bid. The only term that we cannot evaluate directly is  $\mathbf{E}[c_i | \tilde{W}^2 - W^2]$  because we do not know  $c_i$ . However, under the private values assumption, it turns out that we can bound this term using the bidder's third-round bid. We discuss this issue next.

<sup>24</sup>To derive expression (2), note that  $\Pr(\tilde{W}^2) \mathbf{E}[x\mathbf{b}_i^2 - c_i | \tilde{W}^2] = \Pr(\tilde{W}^2 - W^2) \mathbf{E}[x\mathbf{b}_i^2 - c_i | \tilde{W}^2 - W^2] + \Pr(W^2) \mathbf{E}[x\mathbf{b}_i^2 - c_i | W^2]$ .

<sup>25</sup>If  $c_i$  is higher than  $x\mathbf{b}_i^2$ , this will not be a gain, but a loss.

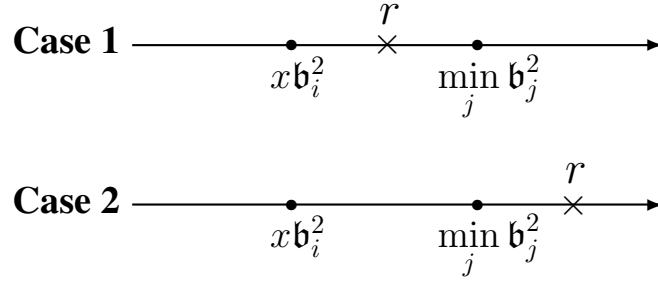


Figure 3: Event  $\tilde{W}^2 - W^2$  includes two possibilities, one in which  $r$  happens to be below the lowest bid in the second round (Case 1) and the other in which  $r$  happens to be above the lowest bid in the second round (Case 2). Note that  $\min_j \mathbf{b}_j^2 < \mathbf{b}_i^2$  in  $\tilde{W}^2 - W^2$ .

Recall that  $\tilde{W}^2 - W^2$  corresponds to the event in which bidder  $i$  wins in the second round with  $x\mathbf{b}_i^2$  but not with  $\mathbf{b}_i^2$ . Event  $\tilde{W}^2 - W^2$  includes two possibilities, one in which  $r$  happens to be below the lowest bid in the second round ( $\{r < \min_j \mathbf{b}_j^2\}$ ) and the other in which  $r$  happens to be above the lowest bid in the second round ( $\{r \geq \min_j \mathbf{b}_j^2\}$ ). Figure 3 depicts the two situations. Note that for Case 1, the auction proceeds to the third round, and we observe  $\mathbf{b}_i^3$ . Hence, we can bound  $c_i$  from above by the observed third-round bid,  $\mathbf{b}_i^3$ . For Case 2, however, the auction ends in the second round, and we do not observe third-round bids.

We now consider how to put bounds on  $c_i$  for Case 2. Note that whether or not the auction proceeds to the third round is, to some extent, independent of the bidders' costs. It depends, in part, on the random realization of  $r$ . Lemma 1 below makes this statement precise; it states that if we have two auctions with the same realizations of  $\{\mathbf{b}_i^1, \min_j \mathbf{b}_j^1, \mathbf{b}_i^2, \min_j \mathbf{b}_j^2\}$ , but one ending in the second round and the other proceeding to the third round, bidder  $i$ 's costs must be the same, on average, in the two auctions. This lemma generalizes the observation that, if bidder  $i$  plays a pure monotone strategy,  $\mathbf{b}_i^1$  is a sufficient statistic for  $c_i$ .<sup>26</sup> The lemma allows us to bound bidder  $i$ 's costs for Case 2 by using the third-round bids conditional on  $\{\mathbf{b}_i^1, \min_j \mathbf{b}_j^1, \mathbf{b}_i^2, \min_j \mathbf{b}_j^2\}$ .

**Lemma 1** *Assume that bidders have private costs  $\mathbf{c} = (c_1, \dots, c_N)$  with density  $g(\mathbf{c})$ ; that*

<sup>26</sup>If bidder  $i$  plays a pure monotone strategy,  $\mathbf{b}_i^1$  is fully revealing about  $i$ 's costs. In particular,  $\mathbf{E}[c_i | \mathbf{b}_i^1, \{r < \min_j \mathbf{b}_j^2\}] = \mathbf{E}[c_i | \mathbf{b}_i^1, \{r \geq \min_j \mathbf{b}_j^2\}]$ . If bidder  $i$  plays a mixed strategy, we need to condition on  $\{\mathbf{b}_i^1, \min_j \mathbf{b}_j^1, \mathbf{b}_i^2, \min_j \mathbf{b}_j^2\}$ .

the secret reserve price,  $r$ , is distributed  $F_r(\cdot)$ ; and that  $\mathbf{c} \perp r$ . Then,

$$\begin{aligned}
& \mathbf{E}[c_i | (\tilde{W}^2 - W^2) \cap \{r \geq \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2] \\
&= \mathbf{E}[c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2] \\
&= \mathbf{E}[c_i | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2].
\end{aligned} \tag{3}$$

Moreover,

$$\begin{aligned}
& \mathbf{E}[c_i | (\tilde{W}^2 - W^2) \cap \{r \geq \min_j \mathfrak{b}_j^2\}] \\
&\leq \mathbf{E}[h(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) | (\tilde{W}^2 - W^2) \cap \{r \geq \min_j \mathfrak{b}_j^2\}],
\end{aligned} \tag{4}$$

where  $h(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) = \mathbf{E}[\mathfrak{b}_i^3 | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2]$ .

**Proof.** See Appendix. ■

The first part of the Lemma states that, conditional on  $\{\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$ , the expected cost of bidder  $i$  for an auction that ends in the second round (the first line of expression (3)) is the same as the expected cost of bidder  $i$  for an auction that goes to the third round (the third line of expression (3)) – under the assumption of private values and  $\mathbf{c} \perp r$ . We argue below that these two assumptions are relatively innocuous in our setting. The second part of the Lemma states that we can bound  $c_i$  in Case 2 using the mean of the observed third-round bids: Note that  $h(\cdot)$  is the expectation of  $\mathfrak{b}_i^3$  conditional on  $\{r < \min_j \mathfrak{b}_j^2\}$ , whereas the expectation of (4) is conditional on Case 2.

The reason that the Lemma is useful is that the right-hand side of expression (4) can be computed using observed data. In particular,  $h(\cdot)$  can be estimated by regressing  $\mathfrak{b}_i^3$  on  $\{\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$  for auctions such that  $\{r < \min_j \mathfrak{b}_j^2\}$ . Note that  $\mathfrak{b}_i^3$  is observed for this set of auctions. We then use the estimated function,  $h(\cdot)$ , to predict the value of  $\mathfrak{b}_i^3$  for Case 2 (i.e., auctions such that  $(\tilde{W}^2 - W^2) \cap \{r \geq \min_j \mathfrak{b}_j^2\}$ ). Note that  $\mathfrak{b}_i^3$  is not observed, but  $\{\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$  are observed for Case 2. The average of the predicted values corresponds to the right-hand side of expression (4).

Before we present our results, we briefly discuss the two assumptions of Lemma 1, namely, that bidders have private values and that the costs and the reserve price are independent. We start with the private values assumption. By assuming that bidders have private values,  $c_i$  becomes constant throughout the three rounds, ensuring that  $\mathfrak{b}_i^3$  is a valid upper bound for  $c_i$  at Round 2. If, instead, bidders have common values, bidders may up-

date their costs in the third round based on the observed lowest second-round bid. If the bidders revise their cost estimates downward in the third round,  $b_i^3$  may no longer be an upper bound on the costs of bidder  $i$  perceived at the time of the second round. For our purposes, whatever assumption – private values or otherwise – that guarantees that  $b_i^3$  is a valid upper bound for  $c_i$  perceived at the time of the second round suffices. The private values assumption is sufficient but not necessary. That is, for mild forms of interdependence, we would still expect  $b_i^3$  to be a valid upper bound on  $c_i$  at Round 2, on average. In this case, our argument would go through without any problems.

Next, we discuss the independence of  $c$  and  $r$ . One might be inclined to argue that the independence assumption is violated based on, for example, the observation that  $c$  and  $r$  are both low for simple jobs (e.g., road paving) and that they are both high for complicated jobs (e.g., bridges). This is *not* necessarily a valid criticism of the independence assumption. In this example, all of the players should be aware that there are two completely different sets of distributions from which  $c$  and  $r$  are drawn, one for road paving and the other for bridges. It is not the case that there is one common set of distributions of  $c$  and  $r$  for both paving and bridges. Conditional on what is common knowledge to the players,  $c$  and  $r$  may very well be independent even in this example.<sup>27</sup> As long as  $c$  and  $r$  are independent conditional on project characteristics that are observable to the *players*, our independence assumption is satisfied.

We are now ready to evaluate  $\Delta\pi$ , the difference in the expected profits from using  $xb_i^2$  instead of  $b_i^2$  in the second round. Using expressions (2) and (4) and the fact that  $b_i^3 > c_i$  for Case 1, we obtain the following bound:

$$\begin{aligned} \Delta\pi \geq & \Pr(\tilde{W}^2 - W^2)\mathbf{E}[xb_i^2|\tilde{W}^2 - W^2] - \Pr(W^2)\mathbf{E}[(1-x)b_i^2|W^2] - \Pr(W^3)\mathbf{E}[b_i^3|W^3] \\ & - \Pr(\{\text{Case 1}\})\mathbf{E}[b_i^3|\{\text{Case 1}\}] \\ & - \Pr(\{\text{Case 2}\})\mathbf{E}[h(b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2)|\{\text{Case 2}\}], \end{aligned} \quad (5)$$

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<sup>27</sup>For example, suppose that bidder costs and the reserve price for paving are given by  $c_i^p = \mu^p + \epsilon_i^p$  and  $r^p = \mu^p + \epsilon^p$ , and similarly for bridges,  $c_i^b = \mu^b + \epsilon_i^b$  and  $r^b = \mu^b + \epsilon^b$ . Suppose, also, that  $\mu^p$  and  $\mu^b$  are constants commonly observed by the bidders, whereas  $\epsilon_i^p$  and  $\epsilon_i^b$  are privately known to the bidders. Then,  $c$  and  $r$  are independent from the perspective of the bidders as long as  $\epsilon_i^p \perp \epsilon^p$  and  $\epsilon_i^b \perp \epsilon^b$  (although they are correlated unconditionally if  $\mu^p \neq \mu^b$ ).

where

$$\begin{aligned} \{\text{Case 1}\} &= (\tilde{W}^2 - W^2) \cap \{r < \min_j \mathbf{b}_j^2\}; \text{ and} \\ \{\text{Case 2}\} &= (\tilde{W}^2 - W^2) \cap \{r \geq \min_j \mathbf{b}_j^2\}. \end{aligned}$$

As explained earlier, we can construct sample analogues of all of the terms on the right-hand side of (5) and evaluate them using data. If the right-hand side is strictly positive, we can reject the null that bidders are playing according to a competitive equilibrium.

We report our results in Table 3. The table presents the estimated values of  $\Delta\pi$  for four different values of  $x$  (99%, 98.5%, 98%, 97.5%) and for three different values of  $\delta$  (1%, 3%, 5%), where  $\delta$  is a measure of the difference between bidder  $i$ 's first-round bid and the lowest first-round bid. Note that inequality (5) still holds even when we condition on the event that bidder  $i$  loses by less than some margin,  $\delta$ , in the first round ( $\{\mathbf{b}_i^1 - \min_j \mathbf{b}_j^1 < \delta \min_j \mathbf{b}_j^1\}$ ). Thus, each cell in Table 3 represents the change in expected profits from using  $x\mathbf{b}_i^2$  instead of  $\mathbf{b}_i^2$  for bidders who lose in the first round by less than  $\delta$ .

Note that all of the cells in Table 3 are positive (and statistically significant at 95%, except for  $(x, \delta) = (98.5\%, 1\%)$ ), implying that firms would be able to increase expected profits by decreasing their second-round bids by a small margin. In terms of magnitude, the numbers seem quite large, considering how loose our inequality is. For example, looking at  $(x, \delta) = (97.5\%, 1\%)$ , we see that the bidder can increase its expected profits by nearly two million yen, on average, by decreasing its second-round bids by 2.5%. Relative to the mean reserve price of 81 million yen for auctions that proceed to the second round (See Table 1), this seems substantial. Our results suggest that bidders are not bidding competitively.

Before concluding this section, we make a few remarks. First, it is possible to conduct the same exercise conditional on various observables, such as year, location, project type, auction category, etc., because inequality (5) holds even when we condition on these observables. In principle, we can even conduct the exercise firm by firm. However, the test is quite data intensive, because it requires many auctions that advance to the third round in order to evaluate  $h(\cdot)$ . The number of auctions in our dataset that reach the third round is not enough for us to apply this test firm by firm. In Section 6, we construct an alternative test that is less data intensive, and we apply it to each firm.

Second, the unconditional test is stronger than the conditional test in some ways. If inequality (5) is violated unconditionally, then it must be violated conditionally for some of the observables.



	Mean $\Delta\pi$ (Yen)			
	$x = 99.0\%$	98.5%	98.0%	97.5%
$\delta = 1\%$	1,048,167 (185,337)	1,093,280 (1,021,015)	1,668,034 (584,337)	1,841,455 (434,903)
3%	438,472 (142,612)	686,734 (282,215)	1,078,694 (232,176)	1,307,195 (210,375)
5%	321,697 (101,754)	576,664 (178,501)	909,425 (159,041)	1,081,212 (178,633)

Note: The numbers in parentheses are the standard deviations.

Table 3: Expected gain in profits from using  $xb_i^2$  for bidders that lose by less than  $\delta$  in the first round.

#### 4.4 Discussion of Our Detection Method

We have discussed several ways of detecting collusion in this section. The key underlying idea is to exploit persistence in the identity of the lowest bidder across multiple rounds of a given auction. We now briefly discuss whether our idea is useful even when bidders become aware of our detection strategy.

As a general point, proposing a detection method can be thought of as putting an additional constraint on the pattern of bidding that a ring can safely engage in. Even if bidding rings respond to a new detection method, the method can still serve a useful purpose by making it potentially harder to sustain collusion, lessening the damage due to collusion, or making it easier to detect collusion with existing methods. To illustrate, one simple way for rings to avoid being detected by our method is to decrease their bids so that the auction ends in the first round. This will diminish the damage from collusion (as the bids have to be lowered, on average) and decrease the incentive to collude.

There are other ways to avoid our detection method that do not involve lowering the bids, such as changing the identity of the lowest bidder across rounds or having the lowest bidder bid substantially less than everybody else. These responses are likely to impose substantial costs on the bidding ring or make detection easier by other means. For example, if the bidding ring changed the identity of the lowest bidder from round to round, this would require at least two bidders to incur the cost of estimating the project. These additional costs would reduce the benefits of collusion, potentially making it harder to sustain it. Alternatively, if one bidder submits a bid that is substantially less than everybody else's,

this would be quite suspicious, putting the ring at risk of being detected by other methods.

## 5 Case Study

In this section, we analyze four collusion cases that were implicated by the JFTC during our sample period. The four cases are the bidding rings of: (A) prestressed concrete providers; (B) firms installing traffic signs; (C) builders of bridge upper structures; and (D) floodgate builders.<sup>28</sup> In all of these cases, firms were found to have engaged in activities such as deciding on a predetermined winner for each project and communicating among the members about how each bidder will bid.<sup>29</sup> All of the implicated firms in cases (B), (C) and (D) admitted wrongdoing soon after the start of the investigation, but none of the firms implicated in case (A) admitted any wrongdoing initially, and the case went to trial.<sup>30</sup>

Before we analyze these four cases, we point out one interesting feature of the bidding ring in case (A): According to the ruling in case (A), an internal rule existed among the subset of the ring members operating in the Kansai region, which prescribed that: 1) the predetermined winner should aim to bid below the reserve price in the first round; 2) if the predetermined winner did not bid below the reserve price in the first round, it should submit a second-round bid that is less than some prespecified fraction (e.g., 0.97) of its first-round bid (e.g.,  $b_{i(1)}^2 < 0.97 \times b_{i(1)}^1$ ); and 3) the rest of the ring members should submit second-round bids that are higher than the prespecified fraction of the *predetermined winner's* first-round bid (e.g.,  $b_{i(k)}^2 > 0.97 \times b_{i(1)}^1$  for  $k \geq 2$ ). The prespecified fractions used in the ring were 0.96 for auctions with an expected value less than 100 million yen; 0.97 for auctions with an expected value between 100 million yen and 500 million yen; and 0.975 for auctions expected to be worth more than 500 million yen. One consequence of this internal rule is that we would observe the same lowest bidder in Round 1 and Round 2.

In Figure 4, we plot the winning bid (lowest bid of the concluding round as a percentage of the reserve price) against the calendar date for all auctions in which the winner is a

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<sup>28</sup>See JFTC Recommendation #27-28 (2004) and Ruling #26-27 (2010) for case (A); JFTC Recommendation and Ruling #5-8 (2005) for case (B); JFTC Recommendation and Ruling #12 (2005) for case (C); and JFTC Cease and Desist Order #2-5 (2007) for case (D).

<sup>29</sup>In all of these cases, the ring members took turns being the predetermined winner. The determination of who would be the predetermined winner depended on factors such as whether a given firm had an existing project that was closely related to the one being auctioned and the number of auctions a given firm had won in the recent past.

<sup>30</sup>Out of 20 firms that were initially implicated in case (A), one firm was acquired by another firm, one was acquitted, and the rest of the firms eventually settled with the JFTC after going to trial.

member of one of the implicated bidding rings. We have also drawn a vertical line that corresponds to the “end date” of collusion. The “end date” is the date, according to the JFTC’s ruling, after which the ring members were deemed to have stopped colluding. The date roughly corresponds to the start date of the investigation. Note that in panels (B) and (C) of Figure 4, there exist periods after the collusion end date during which no ring member wins an auction. This reflects the fact that implicated ring members in cases (B) and (C) were banned from participating in public procurement projects for a period of up to 18 months.<sup>31</sup>

Figure 4 shows that for cases (B), (C), and (D), there is a general drop in the winning bid of about 8.3%, 19.5%, and 5.3%, respectively, after the collusion end date. However, there is almost no change in the winning bid for case (A) before and after the end date. Also, it is worth mentioning that, even for cases (B), (C), and (D), there are some auctions in which the winning bid is extremely high after the end date. In fact, about 24.4% of auctions after the end date have a winning bid higher than 95% for cases (B), (C) and (D). While the investigation and the ruling of the JFTC seem to have made collusion harder, it is far from clear whether the prices after the end date are truly at competitive levels. Hence, the price drops that we see in Figure 4 may be a conservative estimate of the effect of collusion. We expand on this point below.

We now examine the second-round bids of  $i(1)$ ,  $i(2)$ , and  $i(3)$  during the period in which the firms were colluding. If the distinctive shapes of the distribution of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  that we found in Section 4 are, indeed, evidence of collusion, we should expect to see the same pattern among the second-round bids of these colluding firms. Figure 5 plots the histogram of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  before the collusion end date for each of the four bidding rings. The samples used for the figure correspond to the set of auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$  for the left column and  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$  for the right column. We see that for all four bidding rings, the histogram of  $\Delta_{12}^2$  is asymmetric around zero, while the histogram of  $\Delta_{23}^2$  is symmetric around zero, as before. Thus, Figure 5 suggests that the distinctive shapes of the distributions of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  are a hallmark of collusive bidding.

We next examine the second-round bids of the ring members, but for auctions occurring after the collusion end date. To the extent that ring members stopped colluding after the end date, we should expect to see  $\Delta_{12}^2$  to lie to the left of zero in a fair number of auctions. Figure 6 plots the histogram of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  for each of the four bidding rings with  $b_{i(2)}^1 -$

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<sup>31</sup>The ring members involved in cases (A) and (D) were banned from bidding in procurement auctions for certain periods in 2010 and 2007, respectively.

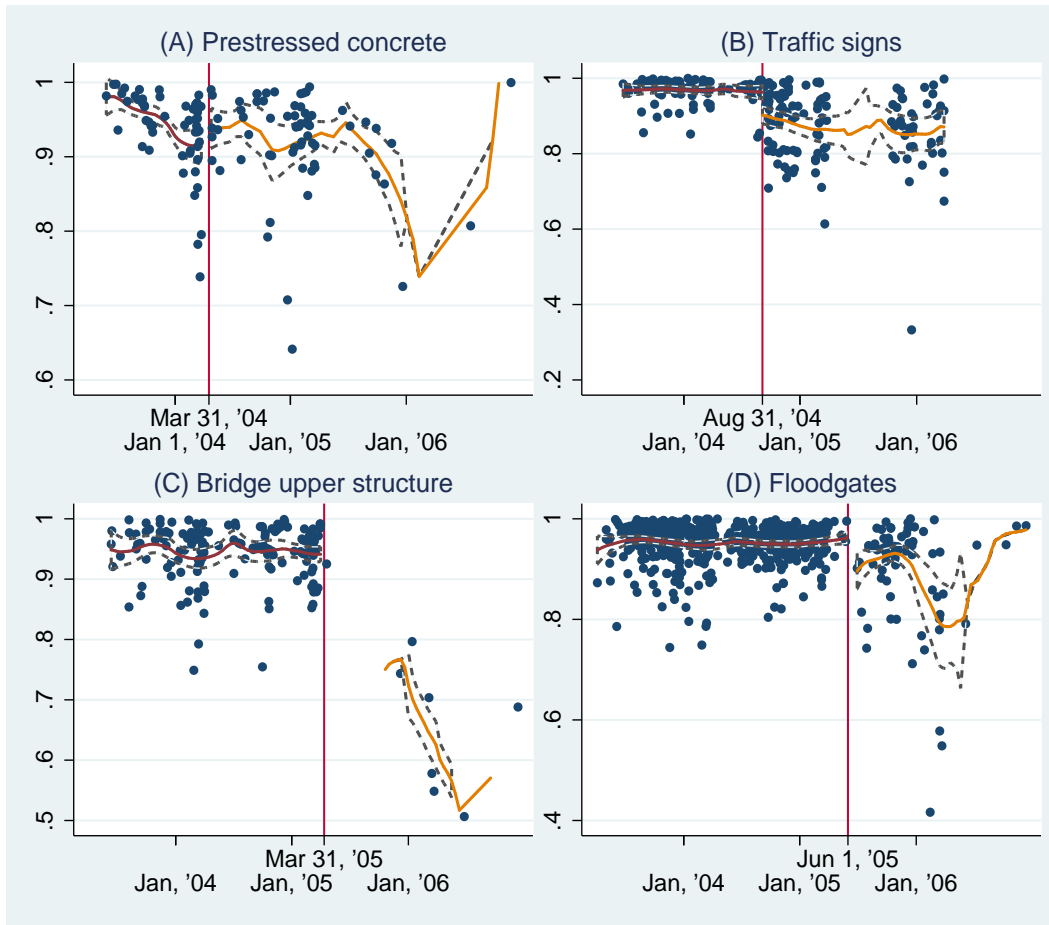


Figure 4: Winning Bid of Auctions in Which the Winner Was Involved in One of the Four Bidding Rings. The horizontal axis corresponds to the calendar date from the beginning of our sample (i.e., April 1, 2003), and the vertical axis corresponds to the winning bid as a percentage of the reserve price. The vertical line in each of the four panels corresponds to the collusion “end date.”

$b_{i(1)}^1 < 5\%$  and  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ . Although the sample sizes are very small, the distributions of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  in Figure 6 are similar to those in Figure 5. That is,  $\Delta_{12}^2$  is distributed to the right of zero, while  $\Delta_{23}^2$  is distributed symmetrically around zero. This may seem to cast doubt on our analysis – why do the distinctive patterns in the distribution of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  persist even after the collusion end date, when firms presumably started behaving competitively?

Our view is that the asymmetry in the distribution of  $\Delta_{12}^2$  should be taken as evidence that the implicated firms were able to continue colluding at least on some auctions, even after the end date. While the bidding rings seem to have changed their behavior around the

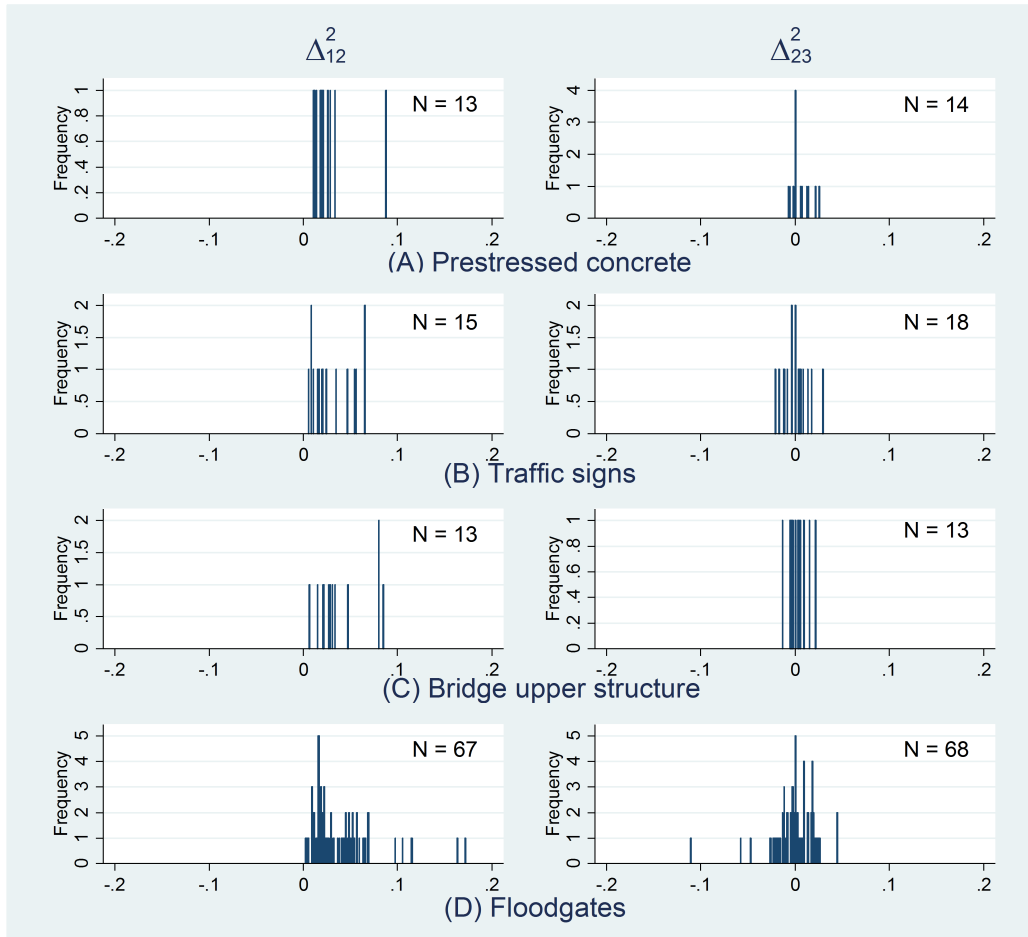


Figure 5: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panels) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panels) Before the Collusion End Date. The left panels correspond to auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ , and the right panels correspond to auctions in which  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ .

time of the end date, – as the drop in the winning bid suggests in Figure 4 – this does not necessarily mean that the firms completely ceased to collude. For example, in the ruling on case (A) issued in 2010, more than five years after the start of the investigation, the judges ordered the ring members to, among other things, take various measures to prevent collusion from recurring.<sup>32</sup> The judges did so because they determined that there were circumstances conducive to collusion even after the end date and that ring members needed

<sup>32</sup>JFTC Ruling #26-27 (2010). In the ruling, the firms were ordered to take preventative measures such as periodic auditing by a legal officer, etc.

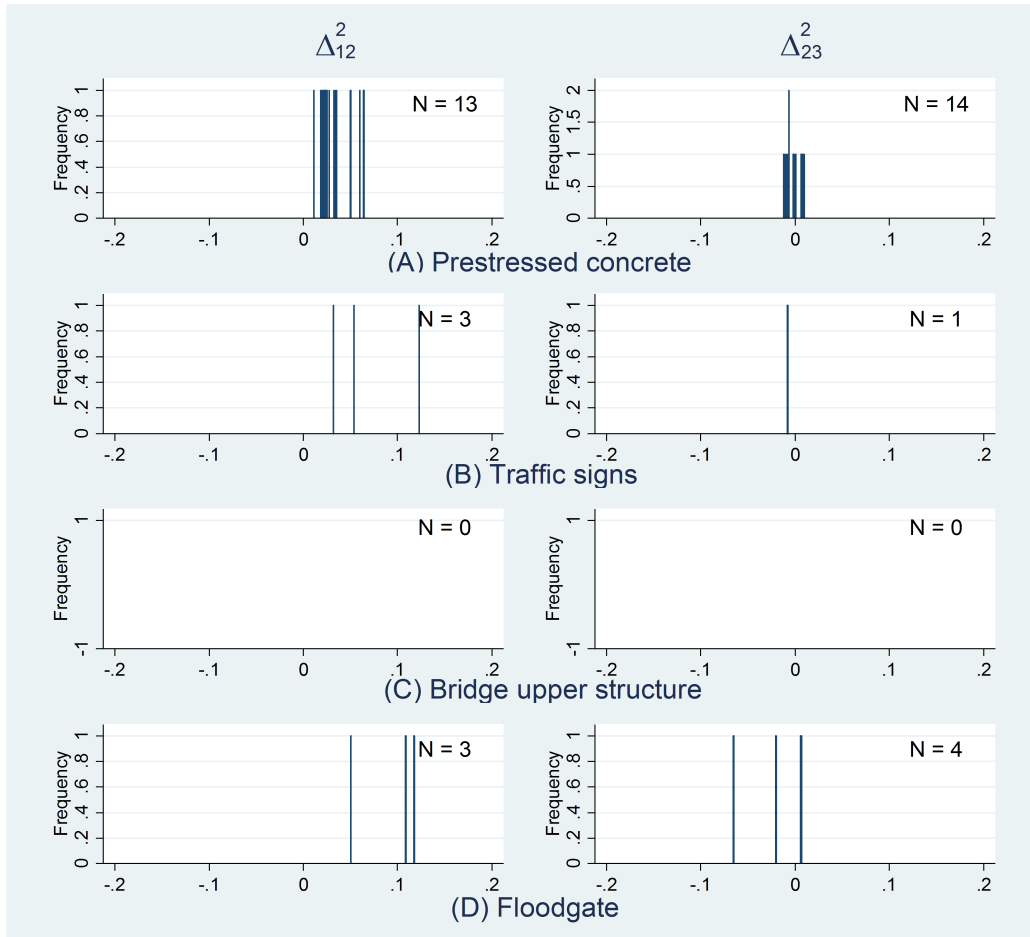


Figure 6: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panels) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panels) After the Collusion End Date. The left panels correspond to auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ , and the right panels correspond to auctions in which  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ .

to take steps to ensure that they did not collude in the future.<sup>33</sup> Moreover, many of the firms implicated in these cases were repeat offenders. For example, one firm involved in case (A) had been found guilty in four previous collusion cases.<sup>34</sup> A number of firms implicated in case (C) were also subsequently charged and found guilty of collusion in a separate case by the JFTC. It seems that being implicated by the JFTC is no guarantee that a firm will behave competitively thereafter; firms may have been able to continue colluding well beyond the end date, at least for some auctions.

<sup>33</sup>See JFTC Rulings #26 (2010) pp.66-68 and #27 (2010) pp.51-54.

<sup>34</sup>JFTC Rulings issued on January 10, 1975; February 25, 1977; July 12, 1977; and June 16, 2000.

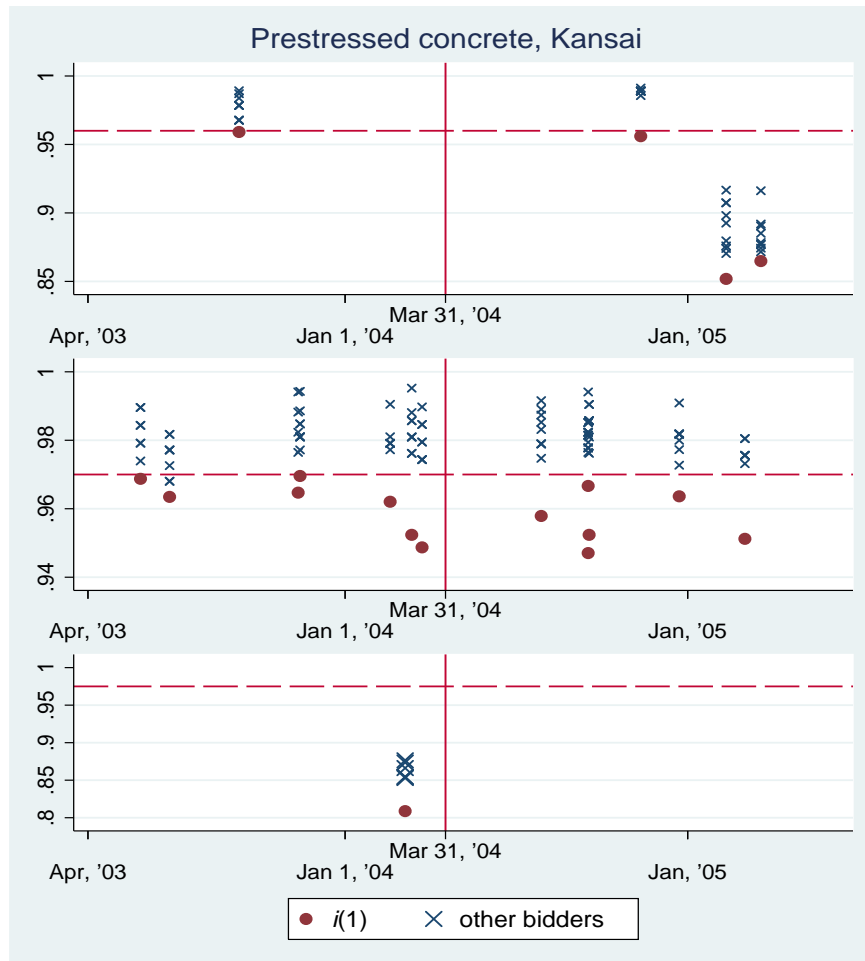


Figure 7: Second-Round Bids of the Ring Members of Kansai Region as a Fraction of the Lowest First-Round Bid. The top panel corresponds to auctions with a reserve price less than 100 million yen; the second panel corresponds to auctions with a reserve price between 100 million and 500 million yen; and the last panel corresponds to auctions with a reserve price above 500 million yen. The horizontal axis corresponds to the calendar date, starting from April 1, 2003. The vertical line at March 31, 2004 corresponds to the end date of the collusion for case (A).

With respect to case (A), there is additional evidence that the ring members continued to collude beyond the end date by following the formula for rebids that we described earlier. Recall that a subset of the prestressed concrete ring members in the Kansai region had a prespecified discount (0.96 for auctions valued at less than 100 million yen; 0.97 for auctions valued between 100 million yen and 500 million yen; and 0.975 for auctions valued at more than 500 million yen) that they used when rebidding in the second round.

Figure 7 plots the second-round bids of the ring members in the Kansai region as a fraction of the lowest first-round bid. The top panel corresponds to auctions with a reserve price below 100 million yen; the middle corresponds to those with a reserve price between 100 and 500 million yen; and the last panel corresponds to those with a reserve price of more than 500 million yen. The horizontal axis in the figure corresponds to the calendar date. The vertical line in each panel corresponds to the collusion end date. Thus, auctions that took place before the end date appear to the left of this line. The circles represent  $b_{i(1)}^2/b_{i(1)}^1$ , and the Xs represent  $b_{i(k)}^2/b_{i(1)}^1$  for  $k \geq 2$ . We have drawn a horizontal line at 0.96 (top panel), 0.97 (middle panel), and 0.975 (bottom panel).

While the top and the bottom panels are not very informative, note that all of  $i(1)$ 's second-round bids in the middle panel of Figure 7 are below 0.97 of  $i(1)$ 's first-round bid. Moreover, the bids of all of the others are above 0.97 of  $i(1)$ 's first-round bid, except for one auction. If we focus on auctions after the collusion end date, the second-round bids of  $i(k)$  ( $k \geq 2$ ) are all above 0.97. The bidding pattern in Figure 7 suggests that bidders continued to use the prespecified discount as the threshold value for submitting second-round bids. It seems quite likely that the ring members were able to maintain collusion even after the end date.

## 6 Detection of Collusive Bidders

In this section, we develop a formal statistical test of collusive behavior that we can apply to each firm. The test is based on the idea we discussed in Section 4.2: the distribution of  $\Delta_{12}^2$  should not be discontinuous at zero under competitive bidding.

**Test Statistic** Recall from Section 4.2 that there is a reasonable amount of variance in the distribution of  $\Delta_{23}^2 (= b_{i(3)}^2 - b_{i(2)}^2)$  even among auctions in which  $i(2)$  and  $i(3)$  submit almost identical first-round bids (See the *right* panels in Figure 1.). To the extent that bids are generated by competitive behavior, this means that there is a reasonable amount of bidder-specific idiosyncrasy with regard to the beliefs over the distribution of the reserve price, risk preference, etc. that induce variance in the second-round bids. This, in turn, implies that  $i(1)$  cannot be outbidding  $i(2)$  in the second round by a small margin all the time under competitive bidding. If  $i(1)$  wins some, it has to lose some, depending on the realization of the bidder-specific idiosyncrasy. Thus, the amount of idiosyncrasy measured by the variance of  $\Delta_{23}^2$  puts a bound on how sharply the distribution of  $\Delta_{12}^2$  can change



around zero. The test statistic that we propose below formalizes this idea by looking for violations of this bound.

We begin by specifying the second-round bids of  $i(2)$  and  $i(3)$  as follows:

$$\begin{aligned} b_{i(2)}^2 &= X + u_2 \\ b_{i(3)}^2 &= X + u_3, \end{aligned}$$

where  $X$  is a common component, and  $u_2, u_3$  are bidder-specific idiosyncratic shocks distributed independently and identically according to  $F_u$ . Given that  $i(2)$  and  $i(3)$  are order statistics, assuming that  $u_2$  and  $u_3$  are identically distributed may seem like a strong assumption. However, as long as we condition on auctions in which the first-round bids of  $i(2)$  and  $i(3)$  are close enough, this specification seems natural: Both  $i(2)$  and  $i(3)$  should have similar cost structures and similar information, which is captured in the common component,  $X$  (In fact, we cannot reject the null that  $u_2$  and  $u_3$  have the same mean in our empirical specification).<sup>35</sup> As we discuss below, we condition on auctions such that the difference in the first-round bids of  $i(2)$  and  $i(3)$  is less than 1% or 5% of the reserve price when we compute the test statistic.

Note that  $X$  is a random variable whose distribution can depend on the object being auctioned, information revealed in the first round, etc. Basically,  $X$  captures all of the common factors between  $i(2)$  and  $i(3)$ . The error terms,  $u_2$  and  $u_3$ , are independent bidder-specific idiosyncrasies that result from differences in the bidders' beliefs over the secret reserve price, heterogeneity in the bidders' risk preferences, etc. We assume that  $u_2$  and  $u_3$  are independent of  $X$ . Now, given that  $\Delta_{23}^2$  is just the difference between  $b_{i(3)}^2$  and  $b_{i(2)}^2$ , we have

$$\begin{aligned} \Delta_{23}^2 &\equiv b_{i(3)}^2 - b_{i(2)}^2 \\ &= u_3 - u_2. \end{aligned}$$

Given our i.i.d. assumptions on  $(u_2, u_3)$ , we can recover  $F_u$  from realizations of  $\Delta_{23}^2$ .

We now consider putting bounds on the distribution of  $\Delta_{12}^2$  using  $F_u$ . Let us denote by

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<sup>35</sup>Below, we use the set of auctions in which  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$  and  $b_{i(3)}^1 - b_{i(2)}^1 < 1\%$  to test for collusion. The means of  $b_{i(3)}^2 - b_{i(2)}^2$  for these two sets of auctions are 0.26% and 0.18% with standard deviations 0.87% and 0.73%. In either case, we cannot reject the null that  $u_2$  and  $u_3$  have the same mean.

$Y$  the second-round bid of  $i(1)$ :

$$b_{i(1)}^2 = Y.^{36}$$

Given that  $i(1)$  has a different information set than all of the other bidders (as well as, perhaps, having different costs), we do not impose any restrictions on the distribution of  $Y$  other than independence with respect to  $(u_2, u_3)$ ; i.e.,  $Y \perp (u_2, u_3)$ . In particular,  $Y$  can have arbitrary correlation with respect to  $X$ .

Note that  $\Delta_{12}^2 = X + u_2 - Y$ , given that  $\Delta_{12}^2 = b_{i(2)}^2 - b_{i(1)}^2$ . Now, we define  $d(t)$  ( $t \in \mathbb{R}^{++}$ ), a measure of how discontinuous the distribution of  $\Delta_{12}^2$  is around zero:

$$d(t) = \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]).$$

$\Pr(\Delta_{12}^2 \in [-t, 0])$  is just the probability that  $\Delta_{12}^2$  falls within  $[-t, 0]$ , and  $\Pr(\Delta_{12}^2 \in [0, t])$  is the probability that  $\Delta_{12}^2$  falls within  $[0, t]$ . Hence,  $d(t)$  is the difference between the probability that  $\Delta_{12}^2$  falls just to the right of zero and the probability that  $\Delta_{12}^2$  falls just to the left of zero.

We can derive a simple bound on  $d(t)$  using  $F_u$  after some algebra,

$$\begin{aligned} d(t) &= \Pr(\Delta_{12}^2 \in [0, t]) - \Pr(\Delta_{12}^2 \in [-t, 0]) \\ &= \int \mathbf{1}_{\{X+u_2-Y \in [0, t]\}} dF_{X,Y}(X, Y) dF_u(u_2) \\ &\quad - \int \mathbf{1}_{\{X+u_2-Y \in [-t, 0]\}} dF_{X,Y}(X, Y) dF_u(u_2) \\ &= \int F_u(Y - X + t) - F_u(Y - X) dF_{X,Y}(X, Y) \\ &\quad - \int F_u(Y - X) - F_u(Y - X - t) dF_{X,Y}(X, Y) \\ &= \int F_u(Y - X + t) + F_u(Y - X - t) - 2F_u(Y - X) dF_{X,Y}(X, Y) \\ &\leq \sup_x \|F_u(x + t) + F_u(x - t) - 2F_u(x)\|, \end{aligned}$$

where the second line uses independence of  $u_2$  with respect to  $X$  and  $Y$ , and  $F_{X,Y}$  is the joint cumulative distribution function of  $X$  and  $Y$ .

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<sup>36</sup>Note that our formulation incorporates specifications such as  $b_{i(1)}^2 = Y + u_1$ .

Our test statistic simply compares  $d(t)$  with the bound derived from  $F_u$ . Define  $\tau(t)$  as

$$\tau(t) \equiv \sup_x \|F_u(x+t) + F_u(x-t) - 2F_u(x)\| - d(t).$$

Given that we can estimate  $F_u$  and  $d(t)$ , we can estimate  $\tau(t)$ . Under the null hypothesis of competitive behavior,  $\tau(t)$  should be nonnegative.

**Detecting Collusive Bidders** We now apply this test to each firm that we observe in the data. In particular, for a given firm, we collect all auctions in which the firm participated. We then estimate  $d(t)$  and  $F_u$  parametrically for each firm, using realizations of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  from the subset of auctions in which 1) the auction proceeded to the second round; and 2) the first-round bids of  $i(2)$  and  $i(3)$  were sufficiently close to each other, i.e.,  $b_{i(3)}^1 - b_{i(2)}^1 < \varepsilon$ .<sup>37</sup> We use a frequency estimator for  $d(t)$  and a maximum likelihood estimator for  $F_u$  by specifying  $F_u$  to be a mean-zero Normal distribution with parameter  $\sigma_u$  ( $u \sim N(0, \sigma_u^2)$ ). While our test statistic can easily accommodate a nonparametric estimate of  $F_u$ , we impose functional form assumptions on  $F_u$  because the number of auctions per firm is not very large. In practice, we estimate  $\tau(t)$  for every firm that participated in at least five auctions that meet the two criteria mentioned above.<sup>38</sup> Given our parametric assumption on  $F_u$ ,  $\tau(t)$  has an asymptotically Normal distribution.

In the top left panel of Figure 8, we plot the estimates of  $\tau(t)$  for each firm for  $t = 1\%$  and  $\varepsilon = 5\%$ . As shown in the panel, the estimated distribution of  $\tau(t)$  lies somewhat to the right of zero, but there is also a substantial mass below zero. Under the null hypothesis of competitive bidding, the value of  $\tau(t)$  should be positive; thus, a negative estimate of  $\tau(t)$  raises concerns about possible collusive behavior. In the top right panel, we plot the  $t$ -statistic for each firm. Again, we find that the estimated  $t$ -statistic is negative for a substantial fraction of firms. In particular, there are 674 firms (out of 3,998 firms) whose  $t$ -statistic is less than  $-1.65$ , which is the one-sided critical value for rejecting the null hypothesis of competitive behavior at the 95% confidence level.<sup>39</sup> In the second row of

<sup>37</sup>Note that we condition on the set of auctions in which the second- and third-lowest bids in the first round are within  $\varepsilon$ , given the assumptions on  $u_2$  and  $u_3$ . Note, also, that we drop auctions if  $\Delta_{23}^2$  is bigger than 30% to make sure that we exclude misrecordings, etc. This biases against finding collusion.

<sup>38</sup>A total of 21,622 construction firms are observed in our analysis, among which 3,998 (3,073) firms participated in at least five auctions that proceeded to the second round with  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$  ( $b_{i(3)}^1 - b_{i(2)}^1 < 1\%$ ).

<sup>39</sup>The set of 674 firms includes 21 firms (out of a total of 92 firms) that were implicated in one of the four bid-rigging cases.

Figure 8, we plot our estimate of  $\tau(t)$  and the  $t$ -statistic for  $t = 2\%$  and  $\varepsilon = 5\%$ . The results are qualitatively similar. For this case, we find that 578 firms have a  $t$ -statistic less than  $-1.65$ .

In the bottom two panels of Figure 8, we repeat the same exercise with  $\varepsilon = 1\%$ . The panels in the third row correspond to  $t = 1\%$ ,  $\varepsilon = 1\%$ , and the bottom panels correspond to  $t = 2\%$ ,  $\varepsilon = 1\%$ . In the third row, there are 403 firms (out of 3,073 firms) whose estimated  $t$ -statistic is less than  $-1.65$ , and in the fourth row, we find 314 firms with an estimated  $t$ -statistic less than  $-1.65$ .

It should be clear from the construction of the test statistic that the value of  $\tau(t)$  should be nonnegative for all values of  $t$  under the null. Hence, we next conduct a joint hypothesis test. In particular, we pick  $t = 1\%$  and  $t = 2\%$  and test whether  $(\tau(1\%), \tau(2\%))$  is jointly nonnegative.<sup>40</sup> Under the joint hypothesis test, we find that we can reject the null for 1,008 firms for  $\varepsilon = 5\%$  (586 firms for  $\varepsilon = 1\%$ ).<sup>41</sup>

To get a sense of the magnitude of our findings, note that the total number of auctions awarded to the 1,008 “suspicious” firms that we identify (in the joint hypothesis test for  $\varepsilon = 5\%$ ) is about 7,600, or close to one fifth of the total number of auctions in our sample. The total award amount of these auctions equals about \$8.6 billion. Given that the four cases we discussed in Section 5 show about an 8.4% average drop in the winning bid after the bidding rings were implicated, we estimate that taxpayers could have saved about \$721 million ( $\$8.6 \text{ billion} \times 8.4\%$ ) in the absence of collusion.<sup>42</sup>

While this is already a large number, note that the total award amount of public construction projects in Japan is about \$200 billion per year, or approximately twenty times the size of our dataset. There is ample reason to believe that collusion is just as rampant in other public construction projects as in those in our dataset, given that many firms in our dataset also participate in other public construction auctions.<sup>43</sup> If we simply scale up

<sup>40</sup>In practice, we estimate the joint (2-dimensional) distribution of  $(\tau(1\%), \tau(2\%))$ . We then simulate 500 draws of  $(2 \times 1)$  random vectors according to the estimated joint distribution. We test whether there are more than 25 (= 5% of 500) draws whose elements are both positive.

<sup>41</sup>The joint hypothesis test for  $\varepsilon = 5\%$  picks out 25 firms out of 92 firms (27 firms for  $\varepsilon = 1\%$ ) that were implicated in one of the four bid-rigging cases.

<sup>42</sup>Note that we are not necessarily overestimating the potential loss to taxpayers by tallying up all auctions – including those that ended in Round 1 – won by suspicious firms. This is because the 8.4% number that we use is derived from comparing the winning bids of all auctions won by the implicated firms before and after the collusion end date (It is the average drop across cases (A)-(D)). To the extent that some of the auctions prior to the collusion end date are competitive, this is reflected in the 8.4% number.

<sup>43</sup>Also, the rules governing procurement in local procurement auctions are very similar to the ones used by the Ministry of Land, Infrastructure, and Transportation.

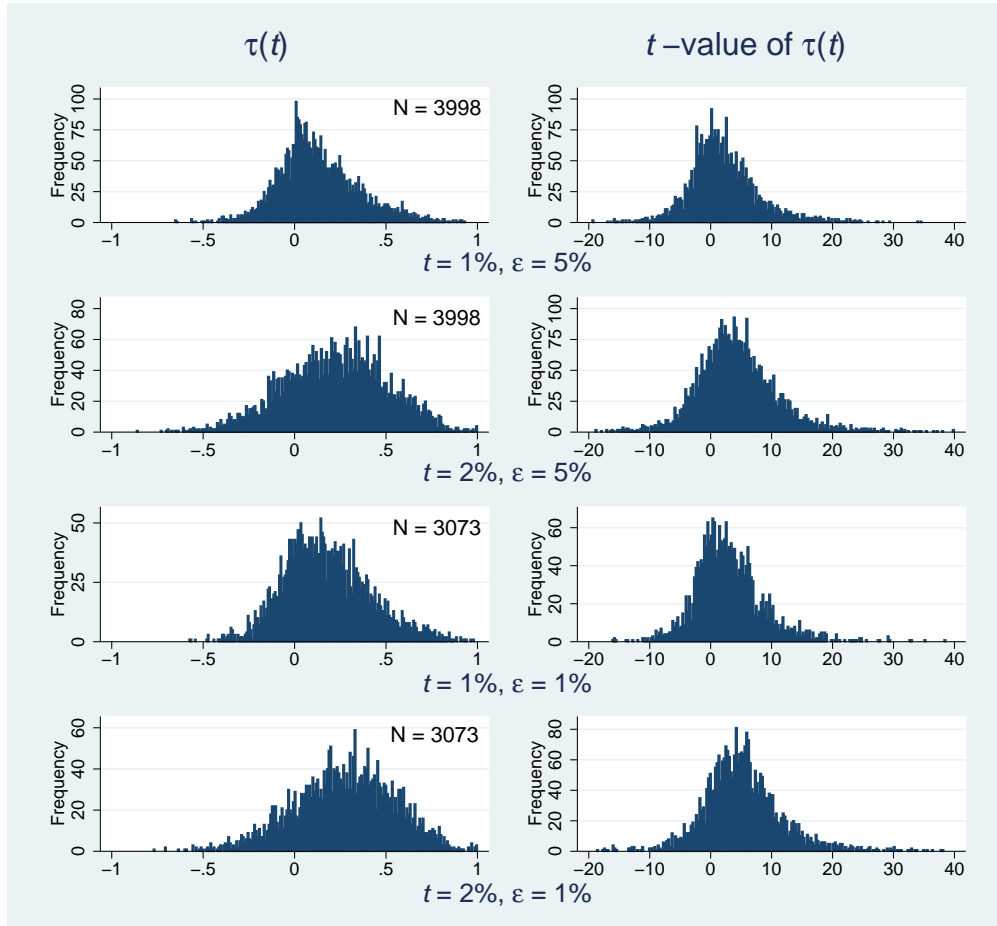


Figure 8: Estimate of  $\tau(t)$  (Left Panel) and  $t$ -Statistic (Right Panel). We estimated  $\tau(t)$  for each firm using only the subset of auctions in which it participated. Top two panels plot the histogram for  $t = 1\%$  and  $t = 2\%$  with  $\varepsilon = 5\%$ . Bottom two panels plot the histogram for  $t = 1\%$  and  $t = 2\%$  with  $\varepsilon = 1\%$ .

our estimates by the size of total public construction spending, our results imply that 4% of total national investment, or 0.85% of GDP, is associated with collusive activity by construction firms. The overall impact of collusion on taxpayers is about \$3.4 billion per year, or about 0.4% of total tax revenue.

## 7 Conclusion

In this paper, we document large-scale collusion among construction firms in Japan using bidding data from government procurement projects. We find evidence of collusion across regions, types of construction projects and time. We then test, for each firm, whether its

bidding behavior is consistent with competitive behavior. Our test identifies about 1,000 “suspicious” firms that won a total of approximately 7,600 auctions, or about one fifth of the total number of auctions during our sample period. Scaling up our estimates by the size of total public construction spending, our results imply that 4% of total national investment, or 0.85% of GDP, is associated with collusive activity by construction firms.

The detection method we propose in this paper is very simple and requires only bid data. While our test is not a definitive proof of collusion, we believe that our method can be useful for law enforcement agencies in identifying possible cases of bid rigging.

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## Appendix

### Proof of Proposition 1

We first prove that the distribution of bidder  $i$ 's cost,  $c_i$ , conditional on  $\{b_i^1, \min_j b_j^1, b_i^2, \min_j b_j^2\}$  is independent of whether or not the auction ends in the second round. Let  $g(\cdot)$  denote the density of  $c_i$  and  $F_r(\cdot)$  denote the distribution function of the reserve price. Note that the probability that the auction proceeds to the second round conditional on  $\min_j b_j^1$  is  $F_r(\min_j b_j^1)$  and that the probability that the auction proceeds to the third round conditional on  $\{\min_j b_j^1, \min_j b_j^2\}$  is  $F_r(\min_j b_j^2)/F_r(\min_j b_j^1)$ . Then,

$$\begin{aligned}
& g(c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{\min_j \mathfrak{b}_j^2 > r\}) \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{\min_j \mathfrak{b}_j^2 > r\})}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{\min_j \mathfrak{b}_j^2 > r\})} \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \overbrace{F_r(\min_j \mathfrak{b}_j^2) / F_r(\min_j \mathfrak{b}_j^1)}^{\text{Pr. go to Rd. 3}}}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times F_r(\min_j \mathfrak{b}_j^2) / F_r(\min_j \mathfrak{b}_j^1)} \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)} \left( = g(c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \right) \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \overbrace{(1 - F_r(\min_j \mathfrak{b}_j^2)) / F_r(\min_j \mathfrak{b}_j^1)}^{\text{Pr. end in Rd. 2}}}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times (1 - F_r(\min_j \mathfrak{b}_j^2)) / F_r(\min_j \mathfrak{b}_j^1)} \\
&= g(c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{\min_j \mathfrak{b}_j^2 \leq r\}).
\end{aligned}$$

Using the fact that the event  $(\tilde{W}^2 - W^2)$  is a coarser partitioning than the partition generated by  $\{\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$ , it is easy to see that the first part of Lemma 1 is true.

We now show the second part of the lemma. From the first part of the lemma,

$$\begin{aligned}
& \mathbf{E}[c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2] \\
&= \mathbf{E}[c_i | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2].
\end{aligned}$$

Replacing  $c_i$  with  $\mathfrak{b}_i^3$  in the second line of the above expression, we obtain

$$\begin{aligned}
& \mathbf{E}[c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2] \\
&\leq \mathbf{E}[\mathfrak{b}_i^3 | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2], \\
&= h(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2).
\end{aligned}$$

Taking expectations on both sides conditional on  $(\tilde{W}^2 - W^2) \cap \{r \geq \min_j \mathfrak{b}_j^2\}$ , we have the second part of the lemma.

## For Online Publication

### Analysis of Collusive Behavior by Region, Auction Category, Project Type, and Time

In this Appendix, we show that the shape of the distributions of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  in Figure 1 is robust to conditioning on region, auction category, project type, and year. We also show that the shape of the distribution is robust to whether or not we normalize the bids by the reserve price. Note that for all of the figures in this section (Figures A.1 - A.5), we set  $\varepsilon$  equal to 5%, i.e., the figures plot auctions in which  $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$  (left panels) or  $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$  (right panels).

#### By Region

Figure A.1 plots the histogram of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  for four of the nine regions of Japan with the largest number of auctions. The regions that we show are Hokkaido, Kanto, Kansai and Chubu, in decreasing order of number of total auctions.

#### By Auction Category

Figure A.2 plots the histogram of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  for each of the four auction categories that we discussed in Section 2. Category 1 corresponds to auctions with the most restrictions on participation, and category 4 corresponds to auctions with the least restrictions.

#### By Project Type

In Figure A.3, we plot the histogram of  $\Delta_{12}^2$  and  $\Delta_{23}^2$  for the four types of projects with the largest number of auctions. The four types of projects are civil engineering, repair and maintenance, paving, and communication equipment, in decreasing order of number of total auctions.

#### By Year

In Figure A.4, we plot the histogram of  $\Delta_{12}^2$  and  $\Delta_{23}^2$ , by year.

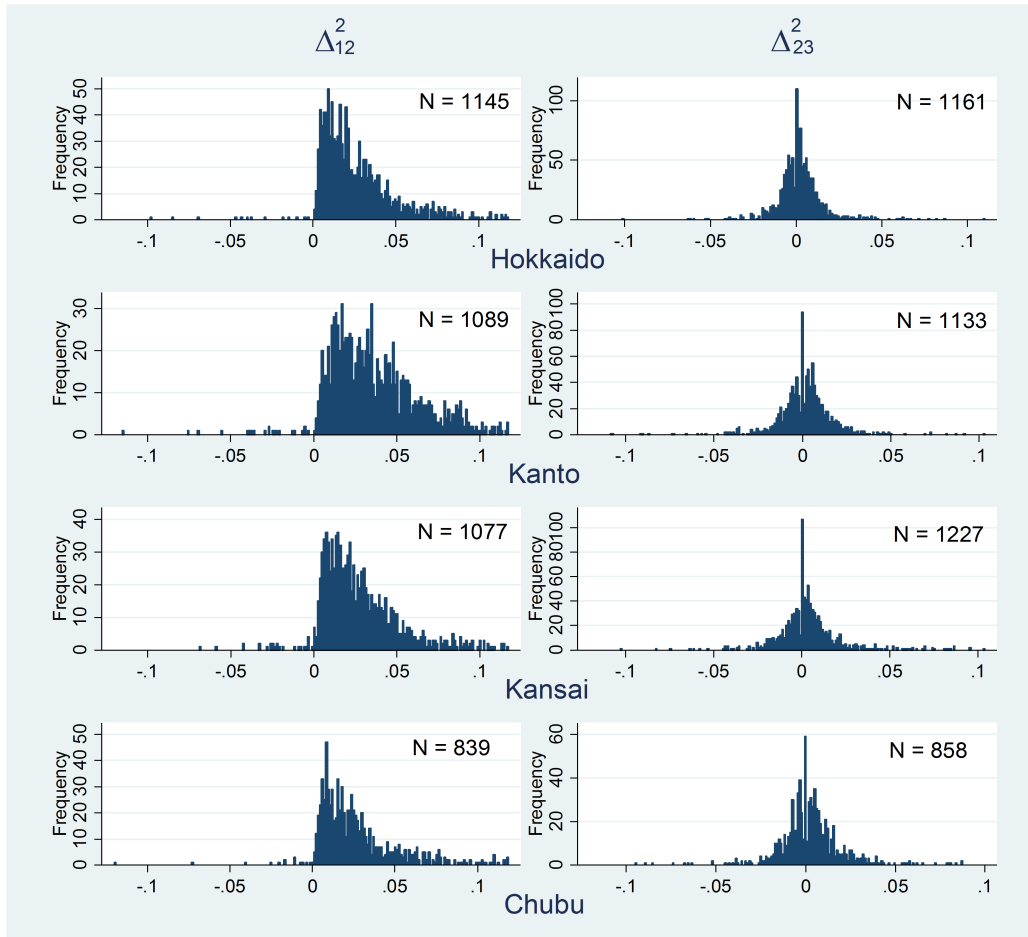


Figure A.1: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panel) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panel), by Region. The left panels plot  $\Delta_{12}^2$  for the set of auctions in which the first-round bids of  $i(1)$  and  $i(2)$  are within 5%. The right panels plot  $\Delta_{23}^2$  for the set of auctions in which the first-round bids of  $i(2)$  and  $i(3)$  are within 5%.

### Raw Bids

Finally, in Figure A.5, we plot the raw difference in the second-round bids without normalizing by the reserve price. The left panels plot the second-round bid differences of  $i(1)$  and  $i(2)$ . The right panels plot the second-round bid differences of  $i(2)$  and  $i(3)$ . The top panels correspond to auctions whose reserve price is between 20 and 22 million yen. The middle and bottom panels correspond to auctions with a reserve price between 60 and 66

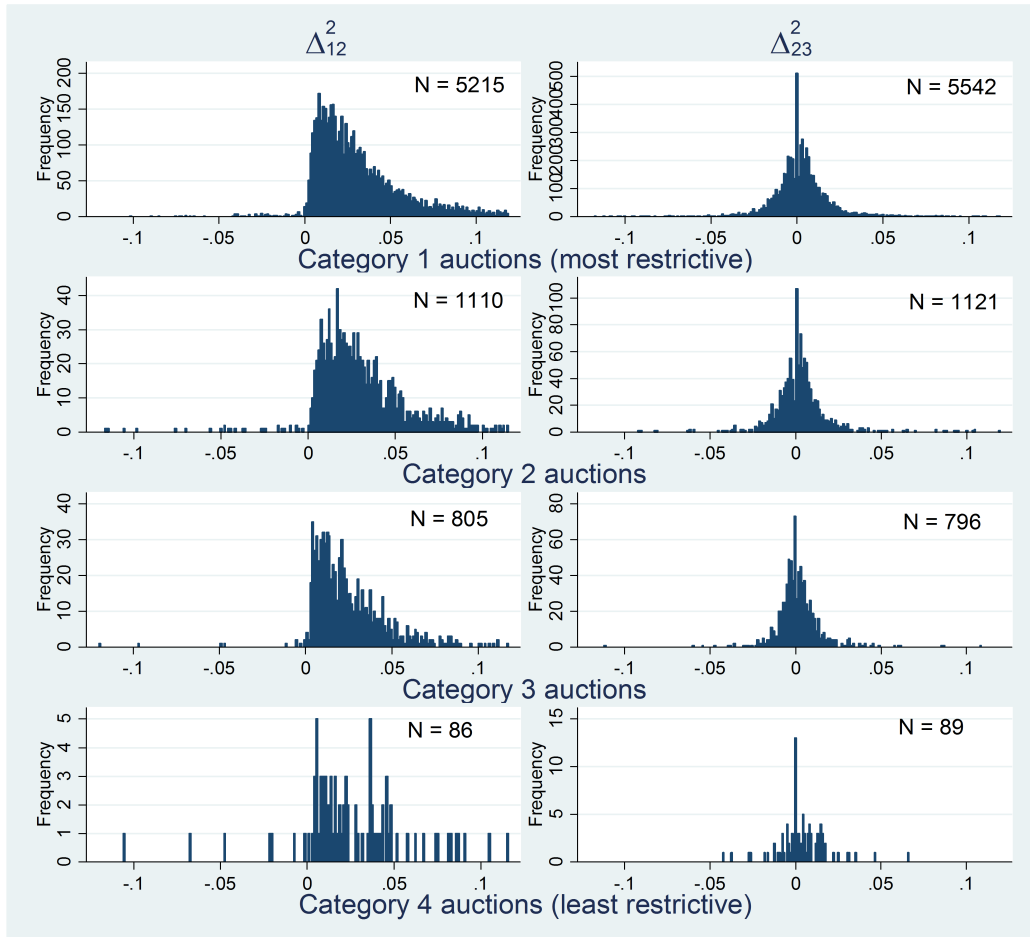


Figure A.2: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panel) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panel), by Auction Category. The left panels plot  $\Delta_{12}^2$  for the set of auctions in which the first-round bids of  $i(1)$  and  $i(2)$  are within 5%. The right panels plot  $\Delta_{23}^2$  for the set of auctions in which the first-round bids of  $i(2)$  and  $i(3)$  are within 5%.

million yen and 90 and 99 million yen, respectively.<sup>44</sup> The auctions in each row roughly correspond to the 25%, 50% and 75% quantiles in terms of project size.

<sup>44</sup>The length of the bandwidth we use (i.e., 2 million, 6 million, and 9 million yen, respectively) is roughly 10% of the reserve price in each row.

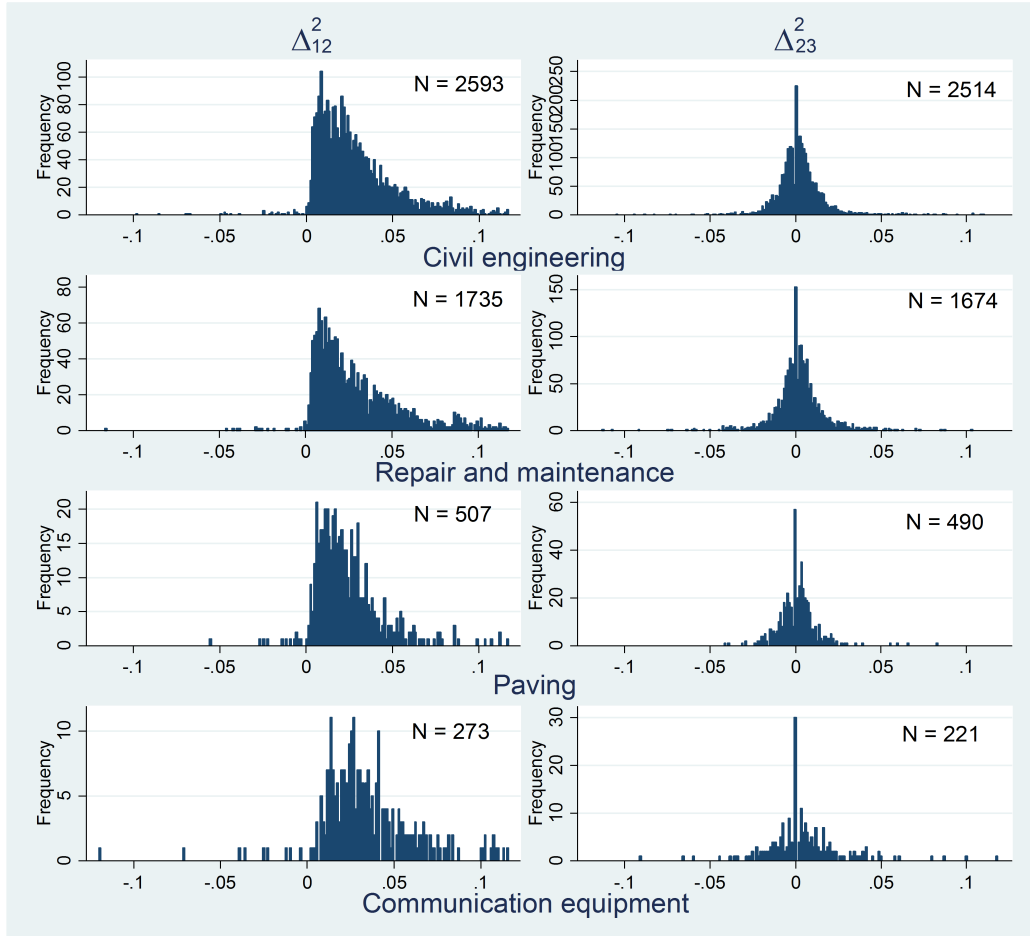


Figure A.3: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panel) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panel), by Project Type. The left panels plot  $\Delta_{12}$  for the set of auctions in which the first-round bids of  $i(1)$  and  $i(2)$  are within 5%. The right panels plot  $\Delta_{23}$  for the set of auctions in which the first-round bids of  $i(2)$  and  $i(3)$  are within 5%.

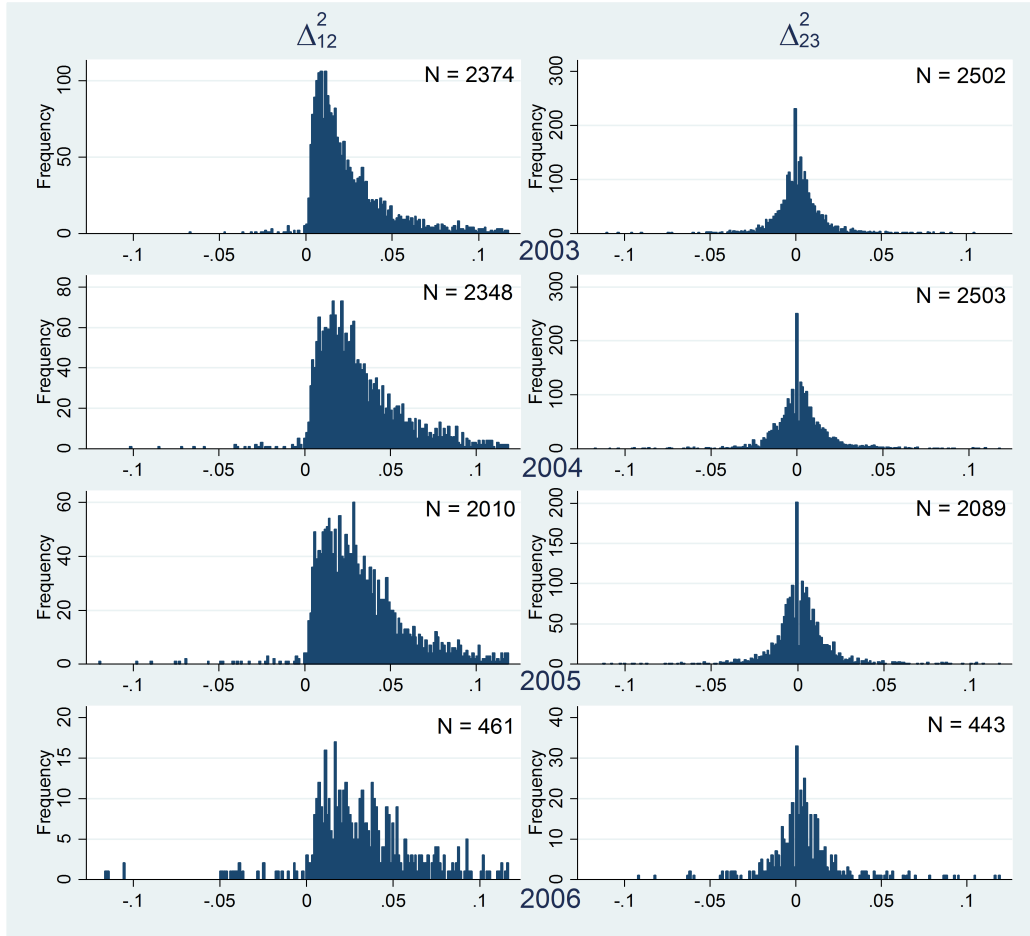


Figure A.4: Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panel) and the Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panel), by Year. The left panels plot  $\Delta_{12}^2$  for the set of auctions in which the first-round bids of  $i(1)$  and  $i(2)$  are within 5%. The right panels plot  $\Delta_{23}^2$  for the set of auctions in which the first-round bids of  $i(2)$  and  $i(3)$  are within 5%.

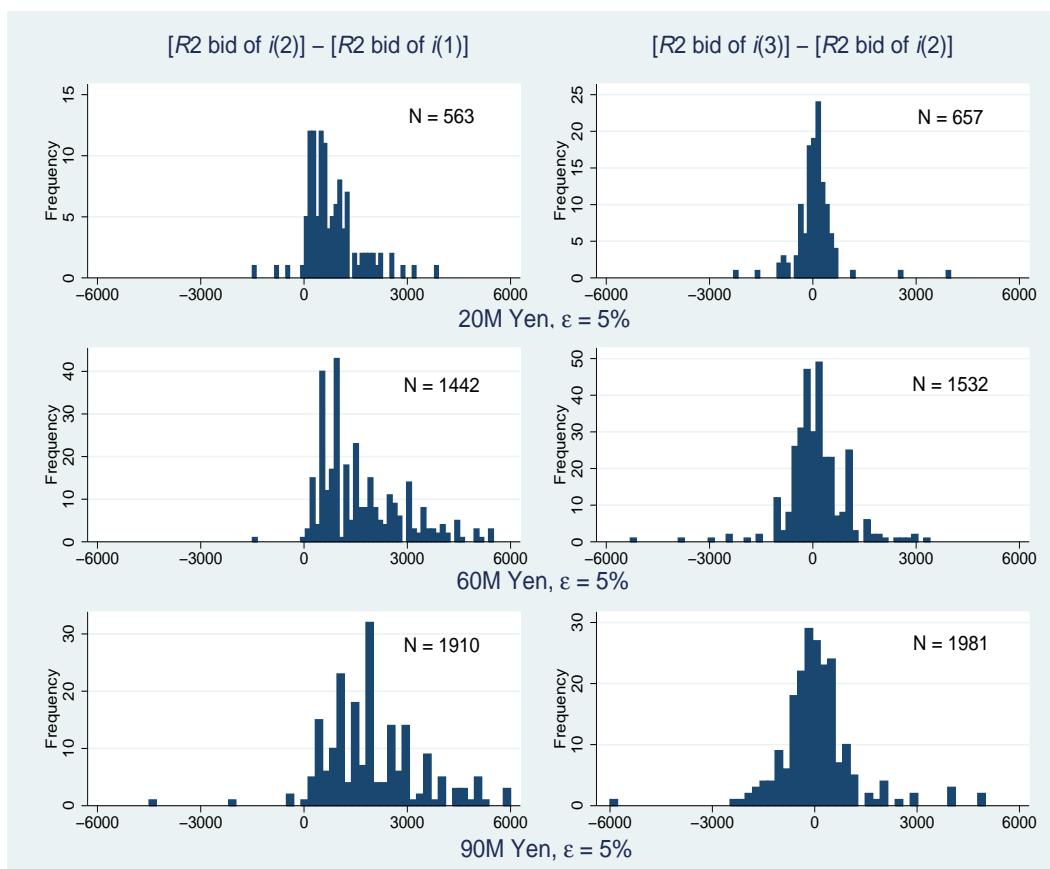


Figure A.5: Raw Difference in the Second-Round Bids of  $i(1)$  and  $i(2)$  (Left Panels) and the Raw Difference in the Second-Round Bids of  $i(2)$  and  $i(3)$  (Right Panels). The left panels plot the raw difference in bids for the set of auctions in which the first-round bids of  $i(1)$  and  $i(2)$  are within 5% of the reserve price. The right panels plot the raw difference in bids for the set of auctions in which the first-round bids of  $i(2)$  and  $i(3)$  are within 5% of the reserve price.